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INTRODUCTION TO HEAVY MESON DECAYS AND CP ASYMMETRIES

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ABSTRACT

These lectures are intended to provide an introduction to heavy meson decays and CP violation. The first lecture contains a brief review of the standard model and how the CKM matrix and CP violation arise, mixing and CP violation in neutral meson systems, and explanation of the cleanliness of the $\sin 2\beta$ measurement. The second lecture deals with the heavy quark limit, some applications of heavy quark symmetry and the operator product expansion for exclusive and inclusive semileptonic B decays. The third lecture concerns with theoretically clean CP violation measurements that may become possible in the future, and some developments toward a better understanding of nonleptonic B decays. The conclusions include a subjective best buy list for the near future.

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Contents

1 Introduction to Flavor Physics: Standard Model Review, Mixing and CP Violation in Neutral Mesons

1.1 Motivation

Flavor physics is the study of interactions that distinguish between the generations. In the standard model (SM), flavor physics in the quark sector and CP violation in flavor changing processes arise from the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. The goal of the B physics program is to precisely test this part of the theory. In the last decade we tested the SM description of electroweak gauge interactions with an accuracy that is an order of magnitude (or even more) better than before. In the coming years tests of the flavor sector and our ability to probe for flavor physics and CP violation beyond the SM may improve in a similar manner.

In contrast to the hierarchy problem of electroweak symmetry breaking, there is no similarly robust argument that new flavor physics must appear near the electroweak scale. Nevertheless, the flavor sector provides severe constraints for model building, and many extensions of the SM do involve new flavor physics near the electroweak scale which may be observable at the B factories. Flavor physics also played an important role in the development of the SM: (i) the smallness of $K^0 - \overline{K}^0$ mixing led to the GIM mechanism and a calculation of the charm mass before it was discovered; (ii) CP violation led to the KM proposal that there should be three generations before any third generation fermions were discovered; and (iii) the large $B^0 - \overline{B}^0$ mixing was the first evidence for a very large top quark mass.

To test the SM in low energy experiments, such as B decays, the main obstacle is that strong interactions become nonperturbative at low energies. The scale dependence of the QCD coupling constant is

$$\alpha_s(\mu) = \frac{\alpha_s(M)}{1 + \dfrac{\alpha_s}{2\pi}\,\beta_0 \ln \dfrac{\mu}{M}} + \dots\,. \tag{1}$$

This implies that at high energies (short distances) perturbation theory is a useful tool. However, at low energies (long distances) QCD becomes nonperturbative, and it is very hard and often impossible to do reliable calculations. There are two scenarios in which making precise predictions is still possible: (i) using extra symmetries of QCD (such as chiral or heavy quark symmetry); or (ii) certain processes are determined by short distance physics. For example, the measurement of $\sin 2\beta$ from $B \to \psi K_S$ is theoretically clean because of CP invariance of the strong interaction, while inclusive B decays are calculable with small model dependence because they are short distance dominated. These will be explained later in detail. Sometimes it is also possible to combine different measurements with the help of symmetries to eliminate uncalculable hadronic physics; this is the case, for example, in $K \to \pi\nu\bar{\nu}$, which is theoretically clean because the form factors that enter this decay are related by symmetries to those measured in semileptonic kaon decay.

These lectures fall short of providing a complete introduction to flavor physics and CP violation, for which there are several excellent books and reviews.[1–8] Rather, I tried to sample topics that illustrate the richness of the field, both in terms of the theoretical methods and the breadth of the interesting measurements. Some omissions might be justified as other lectures covered historical aspects of the field,[9] lattice QCD,[10] physics beyond the standard model,[11] and the experimental status and prospects in flavor physics.[12–16] Unfortunately, the list of references is also far from complete. This writeup follows closely the actual slides shown at the SLAC Summer Institute.

1.2 Standard model — bits and pieces

To define the standard model, we need to specify the gauge symmetry, the particle content, and the pattern of symmetry breaking. The SM gauge group is

$$SU(3)_c \times SU(2)_L \times U(1)_Y \,. \tag{2}$$

Of this, $SU(3)_c$ is the gauge symmetry of the strong interaction, while $SU(2)_L \times U(1)_Y$ corresponding to the electroweak theory. The particle content is defined as three generations of the following representations

$$\begin{aligned} &Q_L(3,2)_{1/6}\,, \qquad u_R(3,1)_{2/3}\,, \qquad d_R(3,1)_{-1/3}\,, \\ &L_L(1,2)_{-1/2}\,, \qquad \ell_R(1,1)_{-1}\,, \end{aligned} \tag{3}$$

where Q_L and L_L are left-handed quark and lepton fields, and u_R, d_R, and ℓ_R are right-handed up-type quarks, down-type quarks, and charged leptons, respectively. The quantum numbers in Eq. (3) are given in the same order as the gauge groups in Eq. (2). Finally the electroweak symmetry, $SU(2)_L \times U(1)_Y$, is broken to electromagnetism, $U(1)_{\rm EM}$, by the vacuum expectation value (VEV) of the Higgs field, $\phi(1,2)_{1/2}$,

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \tag{4}$$

where $v \approx 246\,\text{GeV}$. Once these ingredients of the SM are specified, in principle all particle physics phenomena are determined in terms of 18 parameters, of which 10 correspond to the quark sector (6 masses and 4 CKM parameters).

Some of the most important questions about the SM are the origin of electroweak and flavor symmetry breaking. Electroweak symmetry is spontaneously broken by the dimensionful VEV in Eq. (4), but it is not known yet whether there is an elementary scalar Higgs particle corresponding to ϕ. What we do know, essentially because v is dimensionful, is that the mass of the Higgs (or whatever physics is associated with electroweak symmetry breaking) cannot be much above the TeV scale, since in the absence of new particles, scattering of W bosons would violate unitarity and become strong around a TeV. In contrast, there is no similar argument that flavor symmetry breaking has to do with physics at the TeV scale. If the quarks were massless then the SM would have a global $U(3)_Q \times U(3)_u \times U(3)_d$ symmetry, since the three generations of left handed quark doublets and right handed singlets would be indistinguishable. This symmetry is broken by dimensionless quantities (the Yukawa couplings that give mass to the quarks, see Eq. (7) below) to $U(1)_B$, where B is baryon number, and so we do not know what scale is associated with flavor symmetry breaking. (For the leptons it is not even known yet whether lepton number is conserved; see the discussion below.) One may nevertheless hope that these scales are related, since electroweak and flavor symmetry breaking are connected in many new physics scenarios. There may be new flavor physics associated with the TeV scale, which could have observable consequences, most probably for flavor changing neutral current processes and/or for CP violation.

The most important question in flavor physics is to test whether the SM (i.e., only virtual quarks, W, and Z interacting through CKM matrix in tree and loop diagrams) explain all flavor changing interactions. To be able to answer this question, we need experimental precision, which is being provided by the B factories, and theoretical precision, which can only be achieved in a limited set of processes. Thus, the key processes in this program are those which can teach us about high energy physics with small hadronic uncertainties.

The SM so far agrees with all observed phenomena. Testing the flavor sector as precisely as possible is motivated by the facts that (i) almost all extensions of the SM contain new sources of CP and flavor violation; (ii) the flavor sector is a major constraint for model building, and may distinguish between new physics models; (iii) the observed baryon asymmetry of the Universe requires CP violation beyond the SM. If the scale of new flavor physics is much above the electroweak scale then there will be no observable effects in B decays, and the B factories will make precise SM measurements. However, if there is new flavor physics near the electroweak scale then sizable deviations from the SM predictions are possible, and we could get detailed information on new physics. So the point is not only to measure CKM elements, but to overconstrain the SM predictions by as many "redundant" measurements as possible.

1.2.1 Flavor and CP violation in the SM

The SM is the most general renormalizable theory consistent with the gauge symmetry and particle content in Eqs. (2) and (3). Its Lagrangian has three parts. (The discussion in this section follows Ref. [7].) The kinetic terms are

$$\mathcal{L}_{\rm kin} = -\frac{1}{4}\sum_{\rm groups}(F^a_{\mu\nu})^2 + \sum_{\rm rep's}\overline{\psi}\, i\not{D}\,\psi\,, \tag{5}$$

where $D_\mu = \partial_\mu + ig_s G^a_\mu L^a + igW^b_\mu T^b + ig' B_\mu Y$. Here L_a are the $SU(3)$ generators (0 for singlets, and the Gell-Mann matrices, $\lambda_a/2$, for triplets), T_b are the $SU(2)_L$ generators (0 for singlets, and the Pauli matrices, $\tau_a/2$, for doublets), and Y are the $U(1)_Y$ charges. The $(F^a_{\mu\nu})^2$ terms are always CP conserving. Throughout these lectures we neglect a possible $(\theta_{\rm QCD}/16\pi^2) F_{\mu\nu}\widetilde{F}^{\mu\nu}$ term in the QCD Lagrangian, which violates CP. The constraints on the electron and neutron electric dipole moments imply that the effects of $\theta_{\rm QCD}$ in flavor changing processes are many orders of magnitude below the sensitivity of any proposed experiment (see Ref. [17] for details). The Higgs terms,

$$\mathcal{L}_{\rm Higgs} = |D_\mu\phi|^2 + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2\,, \qquad v^2 = \mu^2/\lambda\,, \tag{6}$$

cannot violate CP if there is only one Higgs doublet. With an extended Higgs sector, CP violation would be possible. Finally, the Yukawa couplings are given by

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\widetilde{\phi}\, u^I_{Rj} - Y^\ell_{ij}\,\overline{L^I_{Li}}\,\phi\,\ell^I_{Rj} + {\rm h.c.}\,, \qquad \widetilde{\phi} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\phi^*\,, \tag{7}$$

where i,j label the three generations, and the superscripts I denote that the quark fields in the interaction basis. To see that CP violation is related to unremovable phases of Yukawa couplings note that the terms

$$Y_{ij}\,\overline{\psi_{Li}}\,\phi\,\psi_{Rj} + Y^*_{ij}\,\overline{\psi_{Rj}}\,\phi^\dagger\,\psi_{Li}\,, \tag{8}$$

become under CP transformation

$$Y_{ij}\,\overline{\psi_{Rj}}\,\phi^\dagger\,\psi_{Li} + Y^*_{ij}\,\overline{\psi_{Li}}\,\phi\,\psi_{Rj}\,. \tag{9}$$

Eqs. (8) and (9) are identical if and only if a basis for the quark fields can be chosen such that $Y_{ij} = Y^*_{ij}$, i.e., that Y_{ij} are real.

After spontaneous symmetry breaking, the Yukawa couplings in Eq. (7) induce mass terms for the quarks,

$$\mathcal{L}_{\rm mass} = -(M_d)_{ij}\,\overline{d^I_{Li}}\,d^I_{Rj} - (M_u)_{ij}\,\overline{u^I_{Li}}\,u^I_{Rj} - (M_\ell)_{ij}\,\overline{\ell^I_{Li}}\,\ell^I_{Rj} + {\rm h.c.}\,, \tag{10}$$

which is obtained by replacing ϕ with its VEV in Eq. (7), and $M_f = (v/\sqrt{2})\,Y^f$, where $f = u, d, \ell$ stand for up- and down-type quarks and charged leptons, respectively. To obtain the physical mass eigenstates, we must diagonalize the matrices M_f. As any complex matrix, M_f can be diagonalized by two unitary matrices, $V_{f\,L,R}$,

$$M_f^{\rm diag} \equiv V_{fL}\,M_f\,V_{fR}^\dagger\,. \tag{11}$$

In this new basis the mass eigenstates are

$$f_{Li} \equiv (V_{fL})_{ij}\, f^I_{Lj}\,, \qquad f_{Ri} \equiv (V_{fR})_{ij}\, f^I_{Rj}\,. \tag{12}$$

We see that the quark mass matrices are diagonalized by different transformations for u_{Li} and d_{Li}, which are part of the same $SU(2)_L$ doublet, Q_L,

$$\begin{pmatrix} u^I_{Li} \\ d^I_{Li} \end{pmatrix} = (V^\dagger_{uL})_{ij} \begin{pmatrix} u_{Lj} \\ (V_{uL}V^\dagger_{dL})_{jk}\, d_{Lk} \end{pmatrix}. \tag{13}$$

The "misalignment" between these two transformations,

$$V_{\rm CKM} \equiv V_{uL}V^\dagger_{dL}\,, \tag{14}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) matrix.[18,19]

This transformation makes the charged current weak interactions, that arise from Eq. (5), appear more complicated in the new basis

$$-\frac{g}{2}\, \overline{Q^I_{Li}}\, \gamma^\mu\, W^a_\mu\, \tau^a\, Q^I_{Li} + {\rm h.c.} \;\Rightarrow\; -\frac{g}{\sqrt{2}}\, (\overline{u_L},\, \overline{c_L},\, \overline{t_L})\, \gamma^\mu\, W^+_\mu V_{\rm CKM} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + {\rm h.c.}\,, \tag{15}$$

where $W^\pm_\mu = (W^1_\mu \mp W^2_\mu)/\sqrt{2}$. As an exercise, show that the neutral current interactions with the Z^0 remain flavor conserving in the mass basis. (This is actually true in all models with only left handed doublet and right handed singlet quarks.) Thus, in the SM all flavor changing processes are mediated by charged current weak interactions, whose couplings to the six quarks are given by a three-by-three unitary matrix, the CKM matrix.

As an aside, let us discuss briefly neutrino masses. With the particle content given in Eq. (3), it is not possible to write down a renormalizable mass term for neutrinos. Such a term would require the existence of a $\nu_R(1,1)_0$ field, a so-called sterile neutrino. Omitting such a field from Eq. (3) is motivated by the prejudice that it would be unnatural for a field that has no SM gauge interactions (is a singlet under all SM gauge groups) to have mass of the order of the electroweak scale. Viewing the SM as an low energy effective theory, there is a single type of dimension-5 terms made of SM fields that are gauge invariant and give rise to neutrino mass, $\frac{1}{\Lambda_{\rm NP}} Y^\nu_{ij} L_i L_j \phi\phi$, where $\Lambda_{\rm NP}$ is a new physics scale. This term violates lepton number by two units. The suppression of this term cannot be the electroweak scale, $\frac{1}{v}$, instead of $\frac{1}{\Lambda_{\rm NP}}$, because such a term in the Lagrangian cannot be generated from SM fields at arbitrary loop level, or even nonperturbatively. (The reason is that such a mass term violates $B-L$, baryon number minus lepton number, which is an accidental symmetry of the SM that is not anomalous.) The above imply that neutrinos are Majorana fermions, since the mass term couples the field ν_L to $\overline{(\nu_L)^c}$ and not to $\overline{\nu_R}$ [the latter occurs for Dirac fermions, see Eq. (10)]. It can be shown that Y^ν_{ij} has to be a real symmetric matrix.

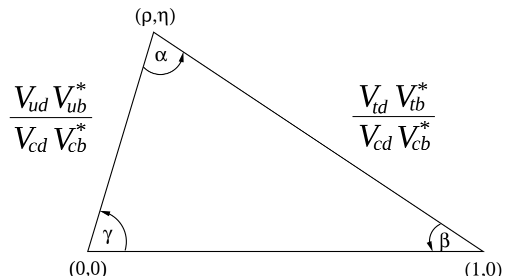


Fig. 1. Sketch of the unitarity triangle.

1.2.2 The CKM matrix and the unitarity triangle

The nine complex entries of the CKM matrix depend on nine real parameters because of unitarity. However, five phases can be absorbed by redefining the quark fields. Thus we are left with four parameters, three mixing angles and a phase. This phase is the only source of CP violation in flavor changing transitions in the SM. A cleaner way to count the number of physical parameters is to note that the two Yukawa matrices, $Y_{i,j}^{u,d}$ in Eq. (7), contain 18 real and 18 imaginary parameters. They break global $U(3)_Q \times U(3)_u \times U(3)_d$ symmetry to $U(1)_B$, so there is freedom to remove 3×3 real and $3\times 6-1$ imaginary parameters. This leaves us with 10 physical quark flavor parameters: 9 real (6 masses and 3 mixing angles) and a complex phase. In the case on N generations, the CKM matrix depends on $N(N-1)/2$ mixing angles and $(N-1)(N-2)/2$ CP violating phases. (In the case of Majorana fermions, one can show following either derivation that there are $N(N-1)/2$ CP violating phases.)

It has been observed experimentally that the CKM matrix has a hierarchical structure, which is well exhibited in the Wolfenstein parameterization,

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \ldots . \tag{16}$$

This form is valid to order λ^4. The small parameter is chosen as the sine of the Cabibbo angle, $\lambda = \sin\theta_C \simeq 0.22$, while A, ρ, and η are order unity. In the SM, the only source of CP violation in flavor physics is the phase of the CKM matrix, parameterized by η. The unitarity of $V_{\rm CKM}$ implies that the nine complex elements of this matrix must satisfy $\sum_k V_{ik}V_{jk}^* = \sum_k V_{ki}V_{kj}^* = \delta_{ij}$. The vanishing of the product of the first and third columns provides a simple and useful way to visualize these constraints,

$$V_{ud}\,V_{ub}^* + V_{cd}\,V_{cb}^* + V_{td}\,V_{tb}^* = 0\,, \tag{17}$$

which can be represented as a triangle (see Fig. 1). Making overconstraining measurements of the sides and angles of this unitarity triangle is one of the best ways to look for new physics.

It will be useful to define two angles in addition to those of the triangle in Fig. 1,

$$\beta \equiv \phi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \qquad \beta_s \equiv \arg\left(-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right), \qquad \beta_K \equiv \arg\left(-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right),$$
$$\alpha \equiv \phi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \qquad \gamma \equiv \phi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right). \tag{18}$$

Here β_s (β_K) is the small angle of the "squashed" unitarity triangle obtained by multiplying the second column of the CKM matrix with the third (first) column, and is of order λ^2 (λ^4). β_s is the phase between B_s mixing and the dominant B_s decays, while β_K is the phase between the charm contribution to K mixing and the dominant K decays. Checking, for example, if β_s is small is an equally important test of the SM as comparing the sides and angles of the triangle in Fig. 1.

To overconstrain the unitarity triangle, there are two very important clean measurements which will reach precisions at the few, or maybe even one, percent level. One is $\sin 2\beta$ from the CP asymmetry in $B \to \psi K_S$, which is becoming the most precisely known angle or side of the unitarity triangle. The other is $|V_{td}/V_{ts}|$ from the ratio of the neutral B_d and B_s meson mass differences, $\Delta m_d/\Delta m_s$. These will be discussed in detail in Sec. 1.6.2 and Sec. 1.5, respectively.

Compared to $\sin 2\beta$ and $|V_{td}/V_{ts}|$, for which both the theory and experiment are tractable, much harder is the determination of another side or another angle, such as $|V_{ub}|$, or α, or γ ($|V_{cb}|$ is also "easy" by these criteria). However, our ability to test the CKM hypothesis in B decays will depend on a third best measurement besides $\sin 2\beta$ and $\Delta m_d/\Delta m_s$ (and on "null observables", which are predicted to be small in the SM). The accuracy of these measurements will determine the sensitivity to new physics, and the precision with which the SM is tested. It does not matter whether it is a side or an angle. What is important is which measurements can be made that have theoretically clean interpretations for the short distance physics we are after.

1.3 CP violation before Y2K

How do we know that CP is violated in Nature? Before the start of the B factories, observations of CP violation came from two sources.

1.3.1 CP violation in the universe

The visible Universe is dominated by matter, and antimatter appears to be much more rare. To quantify this asymmetry one usually compares the number of baryons to the number of photons at the present time. Following the evolution of the universe back toward the big bang, this ratio is related to the asymmetry between quarks and antiquarks at about $t \sim 10^{-6}$ seconds after the big bang, when the temperature was $T \sim 1\,\text{GeV}$,

$$\frac{\#(\text{baryons})}{\#(\text{photons})}\bigg|_{\text{now}} \sim \frac{n_q - n_{\overline{q}}}{n_q + n_{\overline{q}}}\bigg|_{t\sim 10^{-6}\,\text{sec}} \sim 5 \times 10^{-10}\,. \qquad (19)$$

It is usually assumed that at even earlier times the universe probably went through an inflationary phase, which would have washed out any baryon asymmetry that may have been present before inflation. There are three conditions first noted by Sakharov[20] that any theory must satisfy in order to allow for the possibility of dynamically generating the asymmetry in Eq. (19). The theory has to contain: (1) baryon number violating interactions; (2) C and CP violation; and (3) deviation from thermal equilibrium.

The first condition is obvious, and the second is required so that the production rate of left (right) handed quarks and right (left) handed antiquarks may differ. The third condition is needed because in thermal equilibrium the chemical potential for quarks and antiquarks is the same (the CPT theorem implies that the mass of any particle and its antiparticle coincide), and so the production and annihilation rates of quarks and antiquarks would be the same even if the first two conditions are satisfied.

The SM contains all three ingredients, but CP violation is too small (independent of the size of the CKM phase) and the deviation from thermal equilibrium during electroweak phase transition is too small if there is only one Higgs doublet. Detailed analyses show that both of these problems can be solved in the presence of new physics, that must contain new sources of CP violation and have larger deviations from thermal equilibrium than that in the SM. However, for example, the allowed parameter space of the minimal supersymmetric standard model is also getting very restricted to explain electroweak baryogenesis (for details, see: Ref. [21]).

While new physics may yield new CP violating effects observable in B decays, it is possible that the CP violation responsible for baryogenesis only affects flavor diagonal processes, such as electron or neutron electric dipole moments. Another caveat is that understanding the baryon asymmetry may have nothing to do with the electroweak scale; in fact with the observation of large mixing angles in the neutrino sector, leptogenesis[22] appears more and more plausible. The idea is that at a very high scale a lepton-antilepton asymmetry is generated, which is then converted to a baryon asymmetry by $B + L$ violating but $B - L$ conserving processes present in the SM. The lepton asymmetry is due to CP violating decays of heavy sterile neutrinos, that live long enough to decay out of thermal equilibrium. However, the relevant CP violating parameters may or may not be related to CP violation in the light neutrino sector.[23]

1.3.2 CP violation in the kaon sector

Prior to 1964, the explanation of the large lifetime ratio of the two neutral kaons was CP symmetry (before 1956, it was C alone). The argument is as follows. The flavor eigenstates,

$$|K^0\rangle = |\bar{s}d\rangle\,, \qquad |\overline{K}{}^0\rangle = |\bar{d}s\rangle\,, \tag{20}$$

are clearly not CP eigenstates. If CP was a good symmetry, then the states with definite CP would be the following linear combinations

$$|K_{1,2}\rangle = \frac{1}{\sqrt{2}}\Big(|K^0\rangle \pm |\overline{K}{}^0\rangle\Big)\,, \qquad CP|K_{1,2}\rangle = \pm|K_{1,2}\rangle\,. \tag{21}$$

Then only the CP even state could decay into two pions, $K_1 \to \pi\pi$, whereas both states could decay to three pions, $K_{1,2} \to \pi\pi\pi$ (explain why!). Therefore one would expect $\tau(K_1) \ll \tau(K_2)$, in agreement with experimental data, since the phase space for the decay to two pions is much larger than that to three pions. The discovery of $K_L \to \pi\pi$ decay at the 10^{-3} level in 1964 was a big surprise.[24] The "natural" explanation for the observed small CP violation was a new interaction, and, indeed, the superweak model[25] was proposed less than a year after the experimental discovery, whereas the Kobayashi-Maskawa proposal[18] came nine years later (but still before even the charm quark was discovered).

To analyze CP violation in kaon decays, one usually defines the observables

$$\eta_{00} = \frac{\langle \pi^0\pi^0|\mathcal{H}|K_L\rangle}{\langle \pi^0\pi^0|\mathcal{H}|K_S\rangle}\,, \qquad \eta_{+-} = \frac{\langle \pi^+\pi^-|\mathcal{H}|K_L\rangle}{\langle \pi^+\pi^-|\mathcal{H}|K_S\rangle}\,, \tag{22}$$

and the two CP violating parameters,

$$\epsilon_K \equiv \frac{\eta_{00} + 2\eta_{+-}}{3}\,, \qquad \epsilon_K' \equiv \frac{\eta_{+-} - \eta_{00}}{3}\,. \tag{23}$$

To understand these definitions, note that because of Bose statistics the $|\pi\pi\rangle$ final state can only be in isospin 0 (i.e., coming from the $\Delta I = \frac{1}{2}$ part of the Hamiltonian, as the initial state is $I = \frac{1}{2}$) or isospin 2 (i.e., $\Delta I = \frac{3}{2}$) combination [see discussion before Eq. (90)]. Isospin is a symmetry of the strong interactions, to a very good approximation. The decomposition of $|\pi\pi\rangle$ in terms of isospin is

$$\begin{aligned} |\pi^0\pi^0\rangle &= -\sqrt{\frac{1}{3}}\,|(\pi\pi)_{I=0}\rangle + \sqrt{\frac{2}{3}}\,|(\pi\pi)_{I=2}\rangle\,, \\ |\pi^+\pi^-\rangle &= \sqrt{\frac{2}{3}}\,|(\pi\pi)_{I=0}\rangle + \sqrt{\frac{1}{3}}\,|(\pi\pi)_{I=2}\rangle\,. \end{aligned} \tag{24}$$

(In kaon physics often an opposite sign convention is used for $|\pi^0\pi^0\rangle$; Eq. (24) agrees with the Clebsch-Gordan coefficients in the PDG, used in B physics.) Then the isospin amplitudes are defined as

$$\begin{aligned} A_I &= \langle (\pi\pi)_I|\mathcal{H}|K^0\rangle = |A_I|\, e^{i\delta_I}\, e^{i\phi_I}\,, \\ \overline{A}_I &= \langle (\pi\pi)_I|\mathcal{H}|\overline{K}{}^0\rangle = |A_I|\, e^{i\delta_I}\, e^{-i\phi_I}\,, \end{aligned} \tag{25}$$

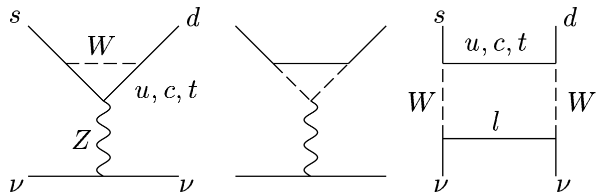


Fig. 2. Diagrams contributing to $K \to \pi\nu\bar{\nu}$ decay (from Ref. [27]).

where $I = 0, 2$, and δ_I and ϕ_I are the strong and weak phases, respectively. It is known experimentally that $|A_0| \gg |A_2|$, which is the so-called $\Delta I = \frac{1}{2}$ rule ($|A_0| \simeq 22\,|A_2|$).

The definition of ϵ_K in Eq. (23) is chosen such that to leading order in the $\Delta I = \frac{1}{2}$ rule only the dominant strong amplitude contributes, and therefore CP violation in decay gives only negligible contribution to ϵ_K (suppressed by $|A_2/A_0|^2$). The world average is $\epsilon_K = e^{i(0.97\pm0.02)\pi/4}\,(2.28 \pm 0.02) \times 10^{-3}$ [26]. Concerning ϵ'_K, to first order in $|A_2/A_0|$,

$$\epsilon'_K = \frac{\eta_{+-} - \eta_{00}}{3} = \frac{\epsilon_K}{\sqrt{2}} \left[\frac{\langle(\pi\pi)_{I=2}|\mathcal{H}|K_L\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{H}|K_L\rangle} - \frac{\langle(\pi\pi)_{I=2}|\mathcal{H}|K_S\rangle}{\langle(\pi\pi)_{I=0}|\mathcal{H}|K_S\rangle} \right]$$
$$= \frac{i}{\sqrt{2}} \left|\frac{A_2}{A_0}\right| e^{i(\delta_2-\delta_0)}\, \sin(\phi_2 - \phi_0)\,. \qquad (26)$$

A non-vanishing value of ϵ'_K implies different CP violating phases in the two isospin amplitudes. The quantity that is actually measured experimentally is $|\eta_{00}/\eta_{+-}|^2 = 1 - 6\,\mathrm{Re}(\epsilon'_K/\epsilon_K)$. The world average is $\mathrm{Re}(\epsilon'_K/\epsilon_K) = (1.8 \pm 0.4) \times 10^{-3}$ [26].

These two observed CP violating parameters in the K system are at the level expected in the SM. The value of ϵ_K can be described with an $\mathcal{O}(1)$ value of the CKM phase and provides a useful constraint. However, precision tests are not yet possible, as ϵ'_K is notoriously hard to calculate in the SM because of enhanced hadronic uncertainties due to contributions that are comparable in magnitude and opposite in sign. (The measurement of ϵ'_K does provide useful constraints on new physics.)

Precision tests of the SM flavor sector in K decays will come from measurements of $K \to \pi\nu\bar{\nu}$, planned in both the neutral and charged modes. These observables are theoretically clean, but the rates are very small, $\sim 10^{-10}$ (10^{-11}) in $K^\pm$ (K_L) decay. They arise from the diagrams in Fig. 2, with intermediate up-type quarks. Due to the GIM mechanism,[28] the rate would vanish in the limit where the up, charm, and top quarks had the same mass. Therefore each contribution to the amplitude is proportional approximately to m_q^2/m_W^2, and we have schematically

$$A \propto \begin{cases} (\lambda^5\, m_t^2) + i(\lambda^5\, m_t^2) & t\!: \text{CKM suppressed}\,, \\ (\lambda\, m_c^2) + i(\lambda^5\, m_c^2) & c\!: \text{GIM suppressed}\,, \\ (\lambda\, \Lambda_{\mathrm{QCD}}^2) & u\!: \text{GIM suppressed}\,, \end{cases} \qquad (27)$$

where we used the phase convention and parameterization in Eq. (16). Each contribution is either GIM or CKM suppressed. So far two $K^+ \to \pi^+\nu\bar{\nu}$ events have been observed,[29] corresponding to a branching ratio $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = \left(1.57^{+1.75}_{-0.82}\right) \times 10^{-10}$. The decay $K_L \to \pi^0\nu\bar{\nu}$ is even cleaner than the charged mode because the final state is CP even,[30] and therefore only the imaginary parts in Eq. (27) contribute, where the charm contribution is negligible and the top contribution is a precisely calculable short distance process. (For a more detailed discussion, see Ref. [15].)

1.4 The B physics program and the present status

In comparison with kaons, the B meson system has several features which makes it well-suited to study flavor physics and CP violation. Because the top quark in loop diagrams is neither GIM nor CKM suppressed, large CP violating effects and large mixing are possible in the neutral B_d and B_s systems, some of which have clean interpretations. For the same reason, a variety of rare decays have large enough branching fractions to allow for detailed studies. Finally, some of the hadronic physics can be understood model independently because $m_b \gg \Lambda_{\rm QCD}$.

The goal of this program is to precisely test the flavor sector via redundant measurements, which in the SM determine CKM elements, but can be sensitive to different short distance physics. New physics is most likely to modify CP violating observables and decays that proceed in the SM via loop diagrams only, such as mixing and rare decays. Therefore, we want to measure CP violating asymmetries, mixing and rare decays, and compare the constraints on the CKM matrix from tree and loop processes.

In the SM all CP violation in flavor changing processes arises from the phase in the CKM matrix. The CKM elements with large (and related) phases in the usual convention are V_{td} and V_{ub}, and all large CP violating phenomena comes from these. In the presence of new physics, many independent CP violating phases are possible; e.g., the phases in B_d and B_s mixing may be unrelated. Then using α, β, γ is only a language, as two "would-be" γ measurements, for example, can be sensitive to different new physics contributions. Similarly, measurements of $|V_{td}|$ and $|V_{ts}|$ from mixing may be unrelated to their values measured in rare decays. Thus, to search for new physics, all possible measurements which have clean interpretations are important; their correlations and the pattern of possible deviations from the SM predictions may be crucial to narrow down type of new physics we are encountering. The B physics program is so broad because independent measurements are the best way to search for new physics.

The allowed regions of ρ and η, imposed by the constraints on ϵ_K, $B_{d,s}$ mixing, $|V_{ub}/V_{cb}|$, and $\sin 2\beta$ are shown in Fig. 3. There is a four-fold discrete ambiguity in the $\sin 2\beta$ measurement. Assuming the SM, this is resolved by $|V_{ub}|$: there is only one allowed region using the $|V_{ub}|$ and $\sin 2\beta$ constraints, whereas there would be four

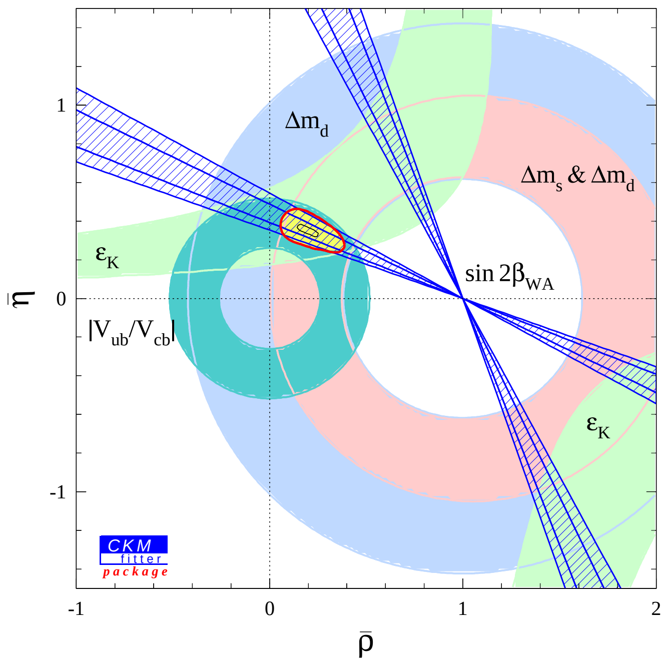


Fig. 3. Present constraints on the CKM matrix (from Ref. [31]).

allowed regions if the $|V_{ub}|$ constraint is removed from the fit.

Figure 3 clearly shows that with the recent precise measurements of $\sin 2\beta$, the CKM picture passed its first real test, and the angle β has become the most precisely known ingredient in the unitarity triangle. Thus, it is very likely that the CKM matrix is the dominant source of CP violation in flavor changing processes at the electroweak scale. This implies a paradigm change in that we can no longer claim to be looking for new physics alternatives of the CKM picture, but to seek corrections to it (a possible exception is still the B_s system). The question is no longer whether the CKM paradigm is right, but whether it is the only observable source of CP violation and flavor change near the electroweak scale.

In looking for modest deviations from the SM, the key measurements are those that are theoretically clean and experimentally doable. Measurements whose interpretation depends on hadronic models cannot indicate unambiguously the presence of new physics. Our ability to test CKM in B decays below the 10% level will depend on the 3rd, 4th, etc., most precise measurements besides β and $|V_{td}/V_{ts}|$ that are used to overconstrain it. (The error of $|V_{td}/V_{ts}|$ is expected to be below 10% once the B_s mass difference is measured, as discussed in Sec. 1.5.) Prospects to measure the $|V_{ub}/V_{cb}|$ side of the UT with small error are discussed in the second lecture, while clean determinations of angles other than β are discussed in the third. Certain observables that are (near) zero in the SM, such as $a_{CP}(B_s \to \psi\phi)$, $a_{CP}(B \to \psi K_S) - a_{CP}(B \to \phi K_S)$, $a_{dir}(B \to s\gamma)$, are also sensitive to new physics and some will be discussed.

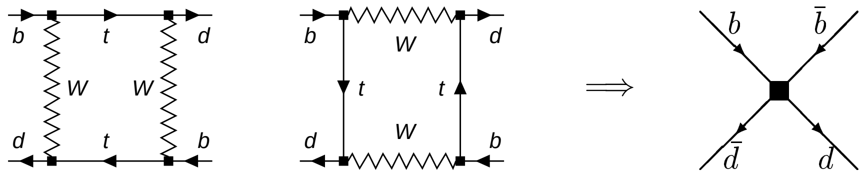


Fig. 4. Left: box diagrams that give rise to the $B^0 - \overline{B}^0$ mass difference; Right: operator in the effective theory below m_W whose B meson matrix element determines Δm_{B_d}.

1.5 B_d and B_s mixing

Similar to the neutral kaon system, there are also two neutral B^0 flavor eigenstates,

$$|B^0\rangle = |\bar{b}\, d\rangle\,, \qquad |\overline{B}^0\rangle = |b\, \bar{d}\rangle\,. \tag{28}$$

The time evolution of a state is described by the Schrödinger equation,

$$i\,\frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}^0(t)\rangle \end{pmatrix} = \Big(M - \frac{i}{2}\,\Gamma\Big) \begin{pmatrix} |B^0(t)\rangle \\ |\overline{B}^0(t)\rangle \end{pmatrix}\,, \tag{29}$$

where the mass mixing matrix, M, and the decay mixing matrix, Γ, are 2×2 Hermitian matrices. CPT invariance implies $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. The heavier and lighter mass eigenstates are the eigenvectors of $M - i\Gamma/2$,

$$|B_{H,L}\rangle = p\,|B^0\rangle \mp q\,|\overline{B}^0\rangle\,, \tag{30}$$

and their time dependence is

$$|B_{H,L}(t)\rangle = e^{-(iM_{H,L} + \Gamma_{H,L}/2)t}\,|B_{H,L}\rangle\,. \tag{31}$$

The solution of the eigenvalue equation is

$$\begin{aligned} (\Delta m)^2 - \frac{1}{4}\,(\Delta\Gamma)^2 &= 4\,|M_{12}|^2 - |\Gamma_{12}|^2\,, \qquad \Delta m\,\Delta\Gamma = -4\,\mathrm{Re}(M_{12}\Gamma_{12}^*)\,, \\ \frac{q}{p} &= -\frac{\Delta m + i\,\Delta\Gamma/2}{2M_{12} - i\,\Gamma_{12}} = -\frac{2M_{12}^* - i\,\Gamma_{12}^*}{\Delta m + i\,\Delta\Gamma/2}\,, \end{aligned} \tag{32}$$

where $\Delta m = M_H - M_L$ and $\Delta\Gamma = \Gamma_L - \Gamma_H$. This defines Δm to be positive, and the choice of $\Delta\Gamma$ is such that it is expected to be positive in the SM (this sign convention for $\Delta\Gamma$ agrees with Ref. [5] and is opposite to Ref. [4]). Note that $M_{H,L}$ ($\Gamma_{H,L}$) are not the eigenvalues of M (Γ). The off-diagonal elements M_{12} and Γ_{12} arise from virtual and on-shell intermediate states, respectively. The contributions to M_{12} are dominated in the SM by box diagrams with top quarks (see Fig. 4), while Γ_{12} is determined by physical states (containing c and u quarks) to which both B^0 and $\overline{B}^0$ can decay.

	$x_q = \Delta m/\Gamma$		$y_q = \Delta\Gamma/\Gamma$		$A_q = 1 - \lvert q/p\rvert^2$	
	theory	data	theory	data	theory	data
B_d	$\mathcal{O}(1)$	≈ 0.75	$y_s\,\lvert V_{td}/V_{ts}\rvert^2$	< 0.2	-0.001	$\lvert A_d\rvert < 0.02$
B_s	$x_d\,\lvert V_{ts}/V_{td}\rvert^2$	> 20	0.1	< 0.4	$-A_d\,\lvert V_{td}/V_{ts}\rvert^2$	—

Table 1. Mixing and CP violation in $B_{d,s}$ mesons. The theory entries indicates rough SM estimates. Data are from the PDG[26] (bounds are 90% or 95% CL).

Simpler approximate solutions can be obtained expanding about the limit $|\Gamma_{12}| \ll |M_{12}|$. This is a good approximation in both B_d and B_s systems. $|\Gamma_{12}| < \Gamma$ always holds, because Γ_{12} stems from decays to final states common to B^0 and $\overline{B}^0$. For the B_s meson the experimental lower bound on Δm_{B_s} implies $\Gamma_{B_s} \ll \Delta m_{B_s}$, and hence $\Gamma_{12}^s \ll \Delta m_{B_s}$ [the theoretical expectation is $\Delta\Gamma_s/\Gamma_s \sim 16\pi^2(\Lambda_{\rm QCD}/m_b)^3$]. For the B_d meson, experiments give $\Delta m_{B_d} \approx 0.75\,\Gamma_{B_d}$. However, Γ_{12}^d arises only due to CKM-suppressed decay channels (giving common final states in B_d^0 and $\overline{B}_d^0$ decay), and so $|\Gamma_{12}^d|/\Gamma_{B_d}$ is expected to be at or below the few percent level (and many experimental analyses assume that it vanishes). In this approximation Eqs. (32) become

$$\Delta m = 2\,|M_{12}|\,, \qquad \Delta\Gamma = -2\,\frac{\mathrm{Re}(M_{12}\Gamma_{12}^*)}{|M_{12}|}\,,$$
$$\frac{q}{p} = -\frac{M_{12}^*}{M_{12}}\left[1 - \frac{1}{2}\,\mathrm{Im}\left(\frac{\Gamma_{12}}{M_{12}}\right)\right]\,, \tag{33}$$

where we kept the second order term in q/p because it will be needed later. Table 1 summarizes the expectations and data for the $B_{d,s}$ systems.

A simple and important implication is that if Γ_{12} is given by the SM, then new physics cannot enhance the $B_{d,s}$ width differences. To see this, rewrite $\Delta\Gamma$ in Eq. (33) as $\Delta\Gamma = 2\,|\Gamma_{12}|\cos[\arg(-M_{12}/\Gamma_{12})]$. In the SM, $\arg(-M_{12}/\Gamma_{12})$ is suppressed by m_c^2/m_b^2 in both $B_{d,s}$ systems (in the B_s system it is further suppressed by the small angle β_s). Consequently, by modifying the phase of M_{12}, new physics cannot enhance $\cos[\arg(-M_{12}/\Gamma_{12})]$, which is near unity in the SM. However, new physics can easily enhance CP violation in mixing, which is suppressed by the small quantity $\sin[\arg(-M_{12}/\Gamma_{12})]$ in the SM, and is especially tiny in the B_s system.

The $B_H - B_L$ mass difference dominated by the box diagrams with top quarks (see Fig. 4) is a short distance process sensitive to physics at high scales (similar to Δm_K). The calculation of Δm_B is a good example of the use of effective theories. The first step is to "match" at the scale of order m_W the box diagrams on the left in Fig. 4 onto the local four-fermion operator, $Q(\mu) = (\bar b_L\gamma_\nu d_L)(\bar b_L\gamma^\nu d_L)$, represented by the diagram on the right. In this step one computes the Wilson coefficient of $Q(\mu = m_W)$.

In the second step, one "runs" the scale of the effective theory down from m_W to a scale around m_b using the renormalization group. In the third step one has to compute the matrix element of $Q(\mu)$ at a scale around m_b. The result is

$$M_{12} = \underbrace{(V_{tb}V_{td}^*)^2}_{\text{WANTED}} \times \underbrace{\frac{G_F^2}{8\pi^2}\,\frac{M_W^2}{m_B}}_{\text{known}} \times \underbrace{S\bigg(\frac{m_t^2}{M_W^2}\bigg)\,\eta_B\, b_B(\mu)}_{\text{calculable perturbatively}} \times \underbrace{\langle B^0|Q(\mu)|\overline{B}^0\rangle}_{\text{nonperturbative}}\,, \qquad (34)$$

where the first term is the combination of CKM matrix elements we want to measure, and the second term contains known factors. The third term contains the matching calculation at the high scale, $S(m_t^2/M_W^2)$ (an Inami-Lim function[32]), and the calculable QCD corrections that occur in running the effective Hamiltonian down to a low scale. It is $\eta_B b_B(\mu)$ that contain the QCD corrections including resummation of the series of leading logarithms, $\alpha_s^n \ln^n(m_W/\mu)$, $\mu \sim m_b$, which is often very important. The last term in Eq. (34) is the matrix element,

$$\langle B^0|Q(\mu)|\overline{B}^0\rangle = \frac{2}{3}\, m_B^2\, f_B^2\, \frac{\widehat{B}_B}{b_B(\mu)}\,, \qquad (35)$$

which is a nonperturbative quantity. It is here that hadronic uncertainties enter, and $f_B^2\, \widehat{B}_B$ has to be determined from lattice QCD. Eq. (34) applies for B_s mixing as well, replacing $V_{td} \to V_{ts}$, $m_{B_d} \to m_{B_s}$, $f_{B_d} \to f_{B_s}$, and $\widehat{B}_{B_d} \to \widehat{B}_{B_s}$.

A clean determination of $|V_{td}/V_{ts}|$ will be possible from the ratio of the B_d and B_s mass differences, $\Delta m_{B_d}/\Delta m_{B_s}$. The reason is that some of the hadronic uncertainties can be reduced by considering the ratio $\xi^2 \equiv (f_{B_s}^2 B_{B_s})/(f_{B_d}^2 B_{B_d})$ which is unity in the flavor $SU(3)$ symmetry limit. Figure 5 shows the preliminary LEP/SLD/CDF combined B_s oscillation amplitude analysis[33] that yields $\Delta m_s > 14.4\,\mathrm{ps}^{-1}$ at 95% CL. Probably B_s mixing will be discovered at the Tevatron, and soon thereafter the experimental error of Δm_s is expected to be at the few percent level.[5] The uncertainty of $|V_{td}/V_{ts}|$ will then be dominated by the error of ξ from lattice QCD. For the last few years the lattice QCD averages[34] have been around $f_{B_s}/f_{B_d} = 1.18 \pm 0.04^{+0.12}_{-0}$ and $B_{B_s}/B_{B_d} = 1.00 \pm 0.03$, in agreement with the chiral log calculation.[35] The last error in the quoted lattice result of f_{B_s}/f_{B_d} reflects an increased appreciation of uncertainties associated with the chiral extrapolation, that may reduce the present results for f_{B_d} but is unlikely to significantly affect f_{B_s}. It is very important to reliably control light quark effects, and to do simulations with three light flavors.*

*Sorting this out reliably may be challenging, since the leading chiral logarithms need not be a good guide to the chiral behavior of quantities involving heavy hadrons. Chiral perturbation theory for processes with heavy hadrons may have a cutoff as low as $500\,\mathrm{MeV}$ instead of $4\pi f_\pi \sim 1\,\mathrm{GeV}$, leading to large "higher order" effects.[36] Using chiral perturbation theory to extrapolate lattice calculations with heavy "light" quarks to the chiral limit may then be questionable.[37]

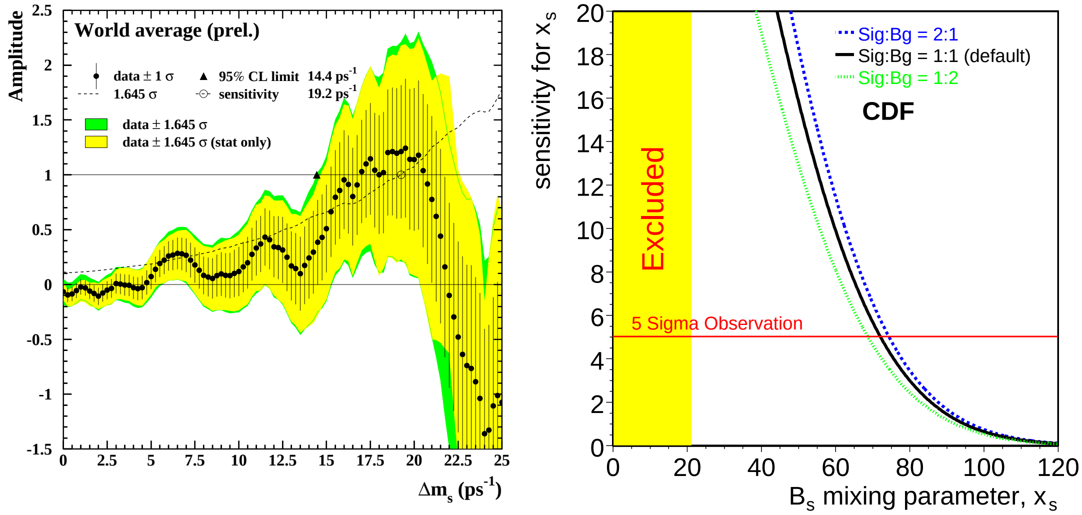


Fig. 5. Left: present B_s oscillation amplitude analysis.[33] Right: projected CDF sensitivity with $2\,\text{fb}^{-1}$ data.[5] Note that $x_s \equiv \Delta m_s/\Gamma_{B_s} \approx \Delta m_s \times 1.46\,\text{ps}$.

1.6 CP violation in the B meson system

1.6.1 The three types of CP violation

***CP* violation in mixing** If CP were conserved, then the mass eigenstates would be proportional to $|B^0\rangle \pm |\overline{B}{}^0\rangle$, corresponding to $|q/p| = 1$ and $\arg(M_{12}/\Gamma_{12}) = 0$. If $|q/p| \neq 1$, then CP is violated. This is called CP violation in mixing, because it results from the mass eigenstates being different from the CP eigenstates. It follows from Eq. (30) that $\langle B_H|B_L\rangle = |p|^2 - |q|^2$, and so if there is CP violation in mixing then the two physical states are not orthogonal. This is clearly a quantum mechanical effect, impossible in a classical system.

The simplest example of this type of CP violation is the semileptonic decay asymmetry of neutral mesons to "wrong sign" leptons,

$$A_{\rm SL}(t) = \frac{\Gamma(\overline{B}{}^0(t)\to \ell^+X) - \Gamma(B^0(t)\to \ell^-X)}{\Gamma(\overline{B}{}^0(t)\to \ell^+X) + \Gamma(B^0(t)\to \ell^-X)} = \frac{1-|q/p|^4}{1+|q/p|^4} = \mathrm{Im}\,\frac{\Gamma_{12}}{M_{12}}\,. \qquad (36)$$

To obtain the right-hand side, we used Eqs. (30) and (31) for the time evolution, and Eq. (33) for $|q/p|$. In kaon decays this asymmetry was recently measured,[38] in agreement with the expectation that it should be equal to $4\,\mathrm{Re}\,\epsilon_K$. In B decays the asymmetry is expected to be[39] $-1.3\times10^{-3} < A_{\rm SL} < -0.5\times10^{-3}$. Figure 6 shows the (weak) constraints on the $\rho-\eta$ plane from the present data on A_{SL}, and what may be achieved by 2005. One can only justify the calculation of $\mathrm{Im}(\Gamma_{12}/M_{12})$ from first principles in the the $m_b \gg \Lambda_{\rm QCD}$ limit, since it depends on inclusive nonleptonic rates. Such a calculation has sizable hadronic uncertainties (by virtue of our limited understanding

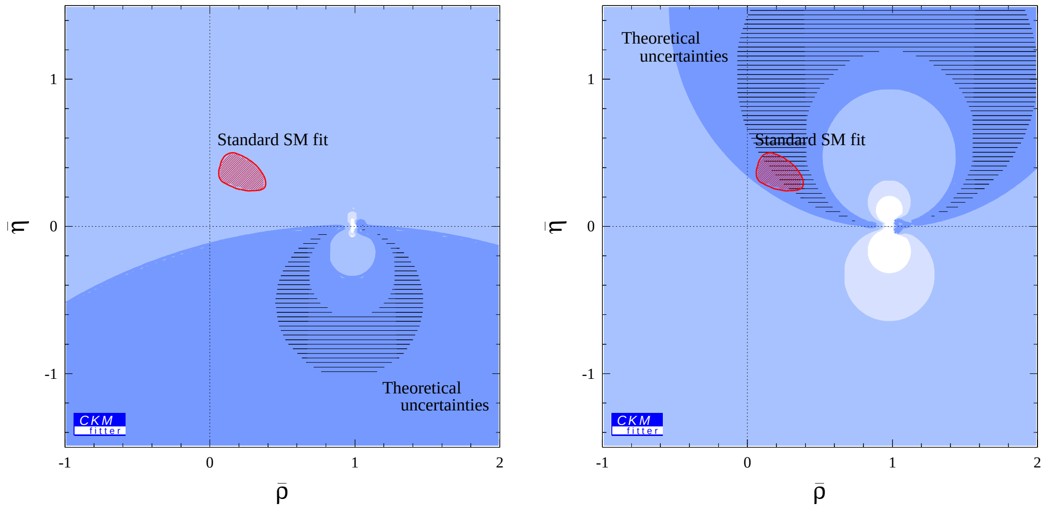


Fig. 6. Left: present constraint from $A_{\rm SL} = (0.2 \pm 1.4) \times 10^{-2}$. Right: constraint that would follow from $A_{\rm SL} = (-1 \pm 3) \times 10^{-3}$ (that may be achieved by 2005). The dark-, medium-, and light-shaded regions have CL $> 90\%$, 32%, and 10%. (From Ref. [39].)

of b hadron lifetimes), an estimate of which is shown by the horizontally stripped regions. However, the constraints on new physics are already interesting,[39] as the m_c^2/m_b^2 suppression of A_{SL} in the SM can be avoided if new physics modifies the phase of M_{12}.

***CP* violation in decay** For most final states f, the $B \to f$ and $\overline{B} \to \overline{f}$ decay amplitudes can, in general, receive several contributions

$$A_f = \langle f|\mathcal{H}|B\rangle = \sum_k A_k\, e^{i\delta_k}\, e^{i\phi_k}, \qquad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle = \sum_k A_k\, e^{i\delta_k}\, e^{-i\phi_k}. \tag{37}$$

There are two types of complex phases which can occur [a similar situation was already encountered in Eq. (25)]. Complex parameters in the Lagrangian which enter a decay amplitude also enter the CP conjugate amplitude but in complex conjugate form. In the SM such weak phases, ϕ_k, only occur in the CKM matrix. Another type of phases are due to absorptive parts of decay amplitudes, and give rise to CP conserving strong phases, δ_k. These arise from on-shell intermediate states rescattering into the desired final state. The individual phases δ_k and ϕ_k are convention dependent, but the phase differences, $\delta_i - \delta_j$ and $\phi_i - \phi_j$, and therefore $|\overline{A}_{\overline{f}}|$ and $|A_f|$, are physical.

Clearly, if $|\overline{A}_{\overline{f}}| \neq |A_f|$ then CP is violated. This is called CP violation in decay, or direct CP violation. Such CP violation can also arise in charged meson and baryon decays, and in B^0 decays in conjunction with the other types. It occurs due to interference between various terms in the decay amplitude, and requires that at least two terms differ both in their strong and in their weak phases,

$$|A|^2 - |\overline{A}|^2 = -4 A_1 A_2 \sin(\delta_1 - \delta_2) \sin(\phi_1 - \phi_2)\,. \tag{38}$$

The only unambiguous observation of direct CP violation to date is $\mathrm{Re}\,\epsilon'_K$ in kaon decay. It can be seen from Eq. (26) that $\mathrm{Im}\,\epsilon'_K$ is not a sign of CP violation in decay, since it may be nonzero even if there is no strong phase difference between the two amplitudes. Note that in B^0 decays different interference type CP violation (see below) in two final states, $\mathrm{Im}\lambda_{f_1} \neq \mathrm{Im}\lambda_{f_2}$, would also be a sign of direct CP violation.

To extract the interesting weak phases from CP violation in decay, one needs to know the amplitudes A_k and their strong phases δ_k. The problem is that theoretical calculations of A_k and δ_k usually have large model dependences. However, direct CP violation can still be very interesting for looking for new physics, especially when the SM prediction is small, e.g., in $b \to s\gamma$.

***CP* violation in the interference between decays with and without mixing** Another type of CP violation is possible when both B^0 and $\overline{B}{}^0$ can decay to the same final state. The simplest example is when this is a CP eigenstate, f_{CP}. If CP is conserved, then not only $|q/p| = 1$ and $|\overline{A}_f/A_f| = 1$, but the relative phase between q/p and $\overline{A}_f/A_f$ also vanishes. It is convenient to define

$$\lambda_{f_{CP}} = \frac{q}{p}\,\frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} = \eta_{f_{CP}}\,\frac{q}{p}\,\frac{\overline{A}_{\overline{f}_{CP}}}{A_{f_{CP}}}\,, \tag{39}$$

where $\eta_{f_{CP}} = \pm 1$ is the CP eigenvalue of f_{CP} [$+1$ (-1) for CP-even (-odd) states]. The second form is useful for calculations, because $A_{f_{CP}}$ and $\overline{A}_{\overline{f}_{CP}}$ are related by CP transformation. If $\mathrm{Im}\lambda_{f_{CP}} \neq 0$ then it is a manifestation of CP violating interference between $B^0 \to f_{CP}$ decay and $B^0 - \overline{B}{}^0$ mixing followed by $\overline{B}{}^0 \to f_{CP}$ decay.

The time dependent asymmetry, neglecting $\Delta\Gamma$, is given by

$$\begin{aligned} a_{f_{CP}} &= \frac{\Gamma[\overline{B}{}^0(t) \to f] - \Gamma[B^0(t) \to f]}{\Gamma[\overline{B}{}^0(t) \to f] + \Gamma[B^0(t) \to f]} \\ &= -\frac{(1 - |\lambda_f|^2)\cos(\Delta m\, t) - 2\,\mathrm{Im}\,\lambda_f \sin(\Delta m\, t)}{1 + |\lambda_f|^2} \\ &\equiv S_f \sin(\Delta m\, t) - C_f \cos(\Delta m\, t)\,. \end{aligned} \tag{40}$$

The last line defines the S and C terms that will be important later on (note that the BELLE notation is $S \equiv \mathcal{S}$ and $C \equiv -\mathcal{A}$). This asymmetry can be nonzero if any type of CP violation occurs. In particular, if $|q/p| \simeq 1$ and $|\overline{A}_f/A_f| \simeq 1$ then it is possible that $\mathrm{Im}\lambda_f \neq 0$, but $|\lambda_f| = 1$ to a good approximation. In both the B_d and B_s systems $|q/p| - 1 < \mathcal{O}(10^{-2})$, so the question is usually whether $|\overline{A}/A|$ is near unity. Even if we cannot compute hadronic decay amplitudes model independently, $|\overline{A}/A| = 1$ is guaranteed if amplitudes with a single weak phase dominate a decay. In such cases we can extract the weak phase difference between $B^0 \to f_{CP}$ and $B^0 \to \overline{B}{}^0 \to f_{CP}$ in a theoretically clean way,

$$a_{f_{CP}} = \mathrm{Im}\lambda_f \sin(\Delta m\, t)\,. \tag{41}$$

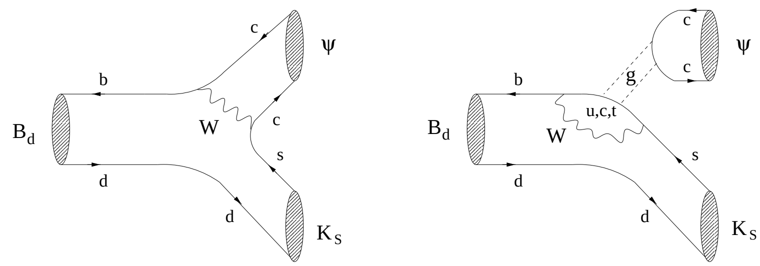


Fig. 7. "Tree" (left) and "Penguin" (right) contributions to $B \to \psi K_S$ (from Ref. [40]).

1.6.2 $\sin 2\beta$ from $B \to \psi K_{S,L}$

This is the cleanest example of CP violation in the interference between decay with and without mixing, because $|\overline{A}/A| - 1 \lesssim 10^{-2}$. Therefore, $\sin 2\beta$ will be the theoretically cleanest measurement of a CKM parameter other than $|V_{ud}|$ (and maybe η from $K_L \to \pi^0 \nu\bar{\nu}$, which, however, is unlikely to be ever measured at the percent level).

There are "tree" and "penguin" contributions to $B \to \psi K_{S,L}$ as shown in Fig. 7. The tree diagram arises from $b \to c\bar{c}s$ transition, while there are penguin contributions with three different combinations of CKM elements,

$$\overline{A}_T = V_{cb}V_{cs}^*\, T_{c\bar{c}s}\,, \qquad \overline{A}_P = V_{tb}V_{ts}^*\, P_t + V_{cb}V_{cs}^*\, P_c + V_{ub}V_{us}^*\, P_u\,. \tag{42}$$

We can rewrite the penguin amplitude using $V_{tb}V_{ts}^* + V_{cb}V_{cs}^* + V_{ub}V_{us}^* = 0$ to obtain

$$\begin{aligned} \overline{A} &= V_{cb}V_{cs}^*\,(T_{c\bar{c}s} + P_c - P_t) + V_{ub}V_{us}^*\,(P_u - P_t) \\ &\equiv V_{cb}V_{cs}^*\, T + V_{ub}V_{us}^*\, P\,, \end{aligned} \tag{43}$$

where the second line defines T and P. We expect $|\overline{A}/A| - 1 < 10^{-2}$, because $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \simeq 1/50$ and model dependent estimates of $|P/T|$ are well below unity. So the amplitude with weak phase $V_{cb}V_{cs}^*$ dominates. The CP asymmetry measures

$$\lambda_{\psi K_{S,L}} = \mp\left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{cb}V_{cs}^*}{V_{cb}^*V_{cs}}\right)\left(\frac{V_{cs}V_{cd}^*}{V_{cs}^*V_{cd}}\right) = \mp e^{-2i\beta}\,, \tag{44}$$

and so $\mathrm{Im}\lambda_{\psi K_{S,L}} = \pm \sin 2\beta$. The first term is the SM value of q/p in B_d mixing, the second is $\overline{A}/A$, and the last one is p/q in the K^0 system. In the absence of $K^0 - \overline{K}^0$ mixing there could be no interference between $\overline{B}^0 \to \psi\overline{K}^0$ and $B^0 \to \psi K^0$.

The first evidence for CP violation outside the kaon sector was the recent BABAR and BELLE measurements[41] of $a_{\psi K}$, whose average, $\sin 2\beta = 0.731 \pm 0.055$, completely dominates the world average[42] already, $\sin 2\beta = 0.734 \pm 0.054$.

1.6.3 $\sin 2\beta$ from $B \to \phi K_{S,L}$

The CP violation in this channel is believed to be a very sensitive probe of new physics. Naively, tree contributions to $b \to s\bar{s}s$ transition are absent, and the penguin contribu-

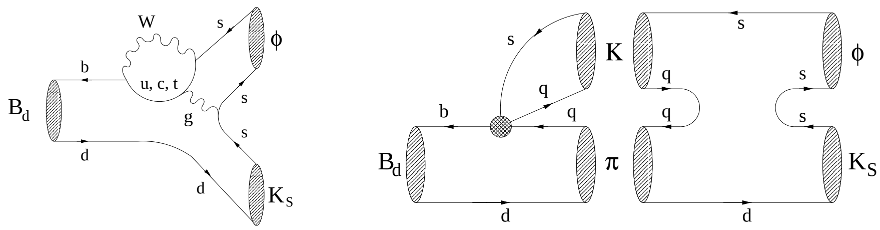


Fig. 8. "Penguin" (left) and "Tree" (right) contributions to $B \to \phi K_S$ (from Ref. [40]).

tions (see Fig. 8) are

$$\overline{A}_P = V_{cb} V_{cs}^* \, (P_c - P_t) + V_{ub} V_{us}^* \, (P_u - P_t) \,. \tag{45}$$

Due to $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \sim \mathcal{O}(\lambda^2)$ and because we expect $|P_c - P_t| \sim |P_u - P_t|$, the $B \to \phi K_S$ amplitude is also dominated by a single weak phase, $V_{cb}V_{cs}^*$. Therefore, $|\overline{A}/A| - 1$ is small, although not as small as in $B \to \psi K_{S,L}$. There is also a "tree" contribution to $B \to \phi K_S$, from $b \to u\bar{u}s$ decay followed by $u\bar{u} \to s\bar{s}$ rescattering, shown in Fig. 8 on the right. This amplitude is also proportional to the suppressed CKM combination, $V_{ub}V_{us}^*$, and it is not even clear how to separate it from "penguin" terms. Unless rescattering provides an enhancement, this should not upset the proximity of $\mathrm{Im}\lambda_{\phi K_S}$ from $\sin 2\beta$. Thus we expect $\mathrm{Im}\lambda_{\phi K_S} = \sin 2\beta + \mathcal{O}(\lambda^2)$ in the SM.

At present $\mathrm{Im}\lambda_{\phi K} = \mathrm{Im}\lambda_{\psi K}$ is violated at the 2.7σ level.[43,44] This is interesting because new physics could enter $\lambda_{\psi K}$ mainly through q/p, whereas it could modify $\lambda_{\phi K}$ through both q/p and $\overline{A}/A$. Note, however, that in the $\eta' K_S$ and $K^+K^-K_S$ channels there is no similarly large deviation from $\sin 2\beta$.[44] The CP asymmetries in $b \to s\bar{s}s$ modes remain some of the best examples that measuring the same angle in several decays sensitive to different short distance physics is one of the most promising ways to look for new physics. This will be very interesting as the errors decrease.

1.7 Summary

- Want experimentally precise and theoretically reliable measurements that in the SM relate to CKM elements, but can probe different short distance physics.
- The CKM picture passed its first real test; we can no longer claim to look for alternatives, but to seek corrections due to new physics (except maybe B_s mixing).
- Very broad program — a lot more interesting as a whole than any single measurement alone; redundancy/correlations may be the key to finding new physics.
- $B_{d,s}$ mixing ($|V_{td}/V_{ts}|$) and $B \to \psi K$ $(\sin 2\beta)$ are "easy", i.e., both theory and experiment are under control; in the next lectures start looking at harder things.

2 Heavy Quark Limit: Spectroscopy, Exclusive and Inclusive Decays

Over the last decade, most of the theoretical progress in understanding B decays utilized the fact that m_b is much larger than $\Lambda_{\rm QCD}$. Semileptonic and rare decays allow measurements of CKM elements important for testing the SM, and are sensitive to new physics. Improving the accuracy of the theoretical predictions increases the sensitivity to new physics. For example, as can be seen from Fig. 3, $|V_{ub}|$ is the dominant uncertainty of the side of the unitarity triangle opposite to the angle β. The constraint from the $K^0 - \overline{K}^0$ mixing parameter ϵ_K is proportional to $|V_{cb}|^4$, and so is the constraint from the $K^+ \to \pi^+\nu\bar\nu$ rate. (The ratio of the $K_L \to \pi^0\nu\bar\nu$ and $K^+ \to \pi^+\nu\bar\nu$ rates is much less sensitive to $|V_{cb}|$.) Most examples in this lecture are related to the determination of $|V_{cb}|$ and $|V_{ub}|$ from exclusive and inclusive semileptonic decays. The same theoretical tools are directly applicable to reducing the hadronic uncertainties in rare decays mediated by flavor changing neutral currents as well.

To believe at some point in the future that a discrepancy is a sign of new physics, model independent predictions are essential. Results which depend on modeling nonperturbative strong interaction effects cannot disprove the Standard Model. Most model independent predictions are of the form,

$$\text{Quantity of interest} = (\text{calculable factors}) \times \bigg[1 + \sum_k (\text{small parameters})^k\bigg], \qquad (46)$$

where the small parameter can be $\Lambda_{\rm QCD}/m_b$, $m_s/\Lambda_{\chi SB}$, $\alpha_s(m_b)$, etc. For the purposes of these lectures we mean by (strong interaction) model independent that the theoretical uncertainty is suppressed by small parameters [so that theorists argue about $\mathcal{O}(1)\times$(small numbers) instead of $\mathcal{O}(1)$ effects]. Still, in most cases, there are theoretical uncertainties suppressed by some $(\text{small parameter})^N$, which cannot be estimated model independently. If the goal is to test the Standard Model, one must assign sizable uncertainties to such "small" corrections not known from first principles.

Throughout the following it should be kept in mind that the behavior of expansions that are formally in powers of $\Lambda_{\rm QCD}/m_b$ can be rather different in practice. (By $\Lambda_{\rm QCD}$ we mean hereafter a generic hadronic scale, and not necessarily the parameter in the running of α_s.) Depending on the process under consideration, the physical scale that determines the behavior of expansions may or may not be much smaller than m_b (and, especially, m_c). For example, f_π, m_ρ, and m_K^2/m_s are all of order $\Lambda_{\rm QCD}$ formally, but their numerical values span an order of magnitude. As it will become clear below, in most cases experimental guidance is needed to decide how well the theory works.

2.1 Heavy quark symmetry and HQET

In hadrons composed of heavy quarks, the dynamics of QCD simplifies. Mesons containing a heavy quark – heavy antiquark pair, $Q\overline{Q}$, form positronium-type bound states, which become perturbative in $m_Q \gg \Lambda_{\rm QCD}$ limit.[45] In heavy mesons composed of a heavy quark, Q, and a light antiquark, $\bar{q}$ (and gluons and $q\bar{q}$ pairs), there are also simplifications in the $m_Q \gg \Lambda_{\rm QCD}$ limit. The heavy quark acts as a static color source with fixed four-velocity, v^μ, and the wave function of the light degrees of freedom (the so-called brown muck) become insensitive to the spin and mass (flavor) of the heavy quark, resulting in heavy quark spin-flavor symmetries.[46]

The physical picture to understand these symmetries is similar to those well-known from atomic physics, where simplifications occur due to the fact that the electron mass, m_e, is much smaller than the nucleon mass, m_N. The analog of flavor symmetry is that isotopes have similar chemistry, because the electrons' wave functions become independent of m_N in the $m_N \gg m_e$ limit. The analog of spin symmetry is that hyperfine levels are almost degenerate, because the interaction of the electron and nucleon spin diminishes in the $m_N \gg m_e$ limit.

The theoretical framework to analyze the consequences of heavy quark symmetry and the corrections to the symmetry limit is the heavy quark effective theory (HQET). One can do a field redefinition to introduce a new field, $h_v(x)$, which annihilates a heavy quark with four-velocity v, and has no dependence on the large mass of the heavy quark,[47]

$$h_v^{(Q)}(x) = e^{i m_Q v\cdot x}\, \frac{1+\not{v}}{2}\, Q(x)\,, \tag{47}$$

where $Q(x)$ denotes the quark field in full QCD. It is convenient to label heavy quark fields by v, because v cannot be changed by soft interactions. The physical interpretation of the projection operator $(1+\not{v})/2$ is that $h_v^{(Q)}$ represents just the heavy quark (rather than antiquark) components of Q. If p is the total momentum of the heavy quark, then the field $h_v^{(Q)}$ carries the residual momentum $k = p - m_Q v \sim \mathcal{O}(\Lambda_{\rm QCD})$. In terms of these fields the QCD Lagrangian simplifies tremendously,

$$\mathcal{L} = \bar{h}_v^{(Q)}\, iv\cdot D\, h_v^{(Q)} + \mathcal{O}\bigg(\frac{1}{m_Q}\bigg), \tag{48}$$

where $D^\mu = \partial^\mu - i g_s T_a A_a^\mu$ is the covariant derivative. The fact that there is no Dirac matrix in this Lagrangian implies that both the heavy quark's propagator and its coupling to gluons become independent of the heavy quark spin. The effective theory provides a well-defined framework to calculate perturbative $\mathcal{O}(\alpha_s)$ and parameterize nonperturbative $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ corrections.

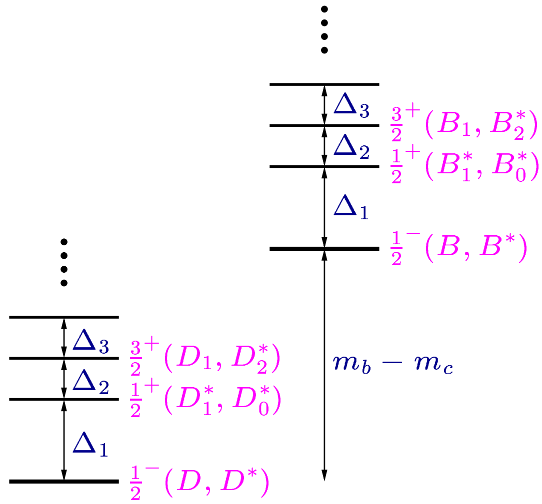


Fig. 9. Spectroscopy of B and D mesons. For each doublet level the spin-parity of the brown muck, $s_l^{\pi_l}$, and the names of the physical states are indicated.

2.1.1 Spectroscopy

The spectroscopy of heavy hadrons simplifies due to heavy quark symmetry because the spin of the heavy quark becomes a good quantum number in $m_Q \to \infty$ limit; i.e., it becomes a conserved quantity in the interactions with the brown muck, $[\vec{s}_Q, \mathcal{H}] = 0$. Of course, the total angular momentum is conserved, $[\vec{J}, \mathcal{H}] = 0$, and therefore the spin of the light degrees of freedom, $\vec{s}_l = \vec{J} - \vec{s}_Q$, also becomes conserved in the heavy quark limit, $[\vec{s}_l, \mathcal{H}] = 0$.

This implies that hadrons containing a single heavy quark can be labeled with s_l, and for any value of s_l there are two (almost) degenerate states with total angular momentum $J_\pm = s_l \pm \frac{1}{2}$. (An exception occurs for baryons with $s_l = 0$, where there is only a single state with $J = \frac{1}{2}$.) The ground state mesons with $Q\bar{q}$ flavor quantum numbers contain light degrees of freedom with spin-parity $s_l^{\pi_l} = \frac{1}{2}^-$, giving a doublet containing a spin zero and spin one meson. For $Q = c$ these mesons are the D and D^*, while $Q = b$ gives the B and B^* mesons.

The mass splittings between the doublets, Δ_i, are of order Λ_{QCD}, and are the same in the B and D sectors at leading order in $\Lambda_{\mathrm{QCD}}/m_Q$, as shown in Fig. 9. The mass splittings within each doublet are of order $\Lambda^2_{\mathrm{QCD}}/m_Q$. This is supported by experimental data: for example, for the $s_l^{\pi_l} = \frac{1}{2}^-$ ground state doublets $m_{D^*} - m_D \approx 140\,\mathrm{MeV}$ while $m_{B^*} - m_B \approx 45\,\mathrm{MeV}$, and their ratio, 0.32, is consistent with m_c/m_b.

As an aside, I cannot resist mentioning a well-known puzzle. Since the ground state vector-pseudoscalar mass splitting is proportional to $\Lambda^2_{\mathrm{QCD}}/m_Q$, we expect $m_V^2 - m_P^2$

to be approximately constant. This argument relies on $m_Q \gg \Lambda_{\rm QCD}$. The data are

$$\begin{aligned} m_{B^*}^2 - m_B^2 &= 0.49\,{\rm GeV}^2\,, \qquad & m_{B_s^*}^2 - m_{B_s}^2 &= 0.50\,{\rm GeV}^2\,, \\ m_{D^*}^2 - m_D^2 &= 0.54\,{\rm GeV}^2\,, \qquad & m_{D_s^*}^2 - m_{D_s}^2 &= 0.58\,{\rm GeV}^2\,, \\ m_\rho^2 - m_\pi^2 &= 0.57\,{\rm GeV}^2\,, \qquad & m_{K^*}^2 - m_K^2 &= 0.55\,{\rm GeV}^2\,. \end{aligned} \tag{49}$$

It is not understood why the light meson mass splittings satisfy the same relation (although this would be expected in the nonrelativistic constituent quark model). There must be something more going on than just heavy quark symmetry, and if this was the only prediction of heavy quark symmetry then we could not say that there is strong evidence that it is a useful idea.

2.1.2 Strong decays of excited charmed mesons

Heavy quark symmetry has implication for the strong decays of heavy mesons as well, because the strong interaction Hamiltonian conserves the spin of the heavy quark and the light degrees of freedom separately.

Excited charmed mesons with $s_l^{\pi_l} = \frac{3}{2}^+$ have been observed. These are the D_1 and D_2^* mesons with spin one and two, respectively. They are quite narrow with widths around $20\,$MeV. This is because their decays to $D^{(*)}\pi$ are in D-waves. An S-wave $D_1 \to D^*\pi$ amplitude is allowed by total angular momentum conservation, but forbidden in the $m_Q \to \infty$ limit by heavy quark spin symmetry.[48] Members of the $s_l^{\pi_l} = \frac{1}{2}^+$ doublet, D_0^* and D_1^*, can decay to $D\pi$ and $D^*\pi$ in S-waves, and therefore these states are expected to be broad. The D_1^* has been observed[49] with a width around $290 \pm 110\,$MeV.[†] The various allowed decays are shown in Fig. 10.

It is possible to make more detailed predictions for the $(D_1, D_2^*) \to (D, D^*)\pi$ decays, since the four amplitudes are related by spin symmetry. The ratios of rates are determined by Clebsch-Gordan coefficients, which are convenient to write in terms of $6j$ symbols,

$$\Gamma(J \to J'\pi) \propto (2s_l+1)(2J'+1) \left| \begin{Bmatrix} L & s_l' & s_l \\ \frac{1}{2} & J & J' \end{Bmatrix} \right|^2 , \tag{50}$$

given in the upper row in Table 2. Since these decays are in $L=2$ partial waves, the phase space depends on the pion momentum as $|p_\pi|^5$ (one can check using Eq. (50)

[†]In the nonrelativistic constituent quark model the $s_l^{\pi_l} = \frac{1}{2}^+$ and $\frac{3}{2}^+$ doublets are $L=1$ orbital excitations (sometimes collectively called D^{**}), and the two doublets arise from combining the orbital angular momentum with the spin of the light antiquark. In the quark model the mass splittings of orbitally excited states vanish as they come from $\langle \vec s_Q \cdot \vec s_{\bar q}\, \delta^3(\vec r\,)\rangle$ interaction. This is supported by the data: $m_{D_2^*} - m_{D_1} = 37\,{\rm MeV} \ll m_{D^*} - m_D$.

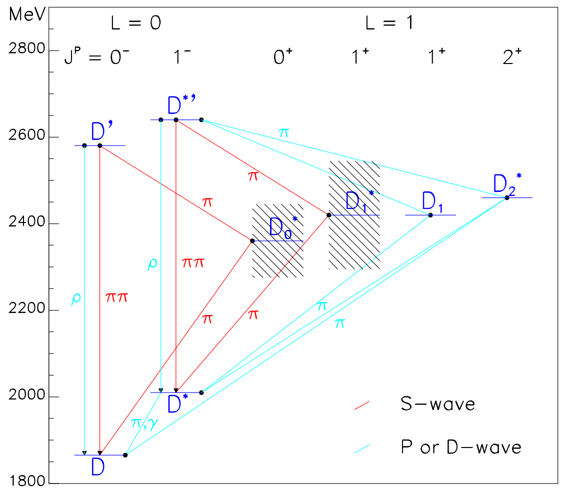


Fig. 10. Spectroscopy and strong decays of D mesons (from Ref. [49]).

$\Gamma(D_1 \to D\pi)$	:	$\Gamma(D_1 \to D^*\pi)$	:	$\Gamma(D_2^* \to D\pi)$	:	$\Gamma(D_2^* \to D^*\pi)$
0	:	1	:	$2/5$	:	$3/5$
0	:	1	:	2.3	:	0.92

Table 2. Ratio of $(D_1, D_2^*) \to (D, D^*)\pi$ decay rates without (upper row) and with (lower row) corrections due to phase space differences (from Ref. [1]).

that the S-wave $D_1 \to D^*\pi$ rate indeed vanishes). This is a large but calculable heavy quark symmetry breaking, which is included in the bottom line of Table 2. It changes the prediction for $\Gamma(D_2^* \to D\pi)/\Gamma(D_2^* \to D^*\pi)$ from $2/3$ to 2.5; the latter agrees well with the data, 2.3 ± 0.6.

The ratio of the D_1 and D_2^* widths works less well: the prediction $1/(2.3+0.9) \simeq 0.3$ is much smaller than the data, $\Gamma(D_1^0)/\Gamma(D_2^{*0}) \simeq 0.7$. The simplest explanation would be that D_1 mixes with the broad D_1^*, due to $\mathcal{O}(\Lambda_{\rm QCD}/m_c)$ spin symmetry violating effects; however, there is no indication of an S-wave component in the $D_1 \to D^*\pi$ angular distribution. The larger than expected D_1 width can be explained with other spin symmetry violating effects.[50] This is important because otherwise it would indicate that we cannot trust the treatment of the charm quark as heavy in other contexts.

2.2 Exclusive semileptonic B decays

Semileptonic and radiative rare decays can be used to determine CKM elements, such as $|V_{cb}|$ and $|V_{ub}|$, and are sensitive probes of new physics. The difficulty is that the hadronic matrix elements that connect exclusive decay rates to short distance weak interaction parameters are not accessible in general theoretically. Important exceptions occur in certain situations due to enhanced symmetries, when some form factors are model independently related to one another, and in the case of $B \to D^*$ decay even the rate is determined at one point in phase space.

2.2.1 $B \to D^{(*)}\ell\bar{\nu}$ decay and $|V_{cb}|$

Heavy quark symmetry is very predictive for $B \to D^{(*)}$ semileptonic form factors. In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, the configuration of the brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin. In the decay of the b quark, the weak current changes suddenly (on a time scale $\ll \Lambda_{\rm QCD}^{-1}$) the flavor $b \to c$, the momentum $\vec{p}_b \to \vec{p}_c$, and possibly flips the spin, $\vec{s}_b \to \vec{s}_c$. In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, because of heavy quark symmetry, the brown muck only feels that the four-velocity of the static color source in the center of the heavy meson changed, $v_b \to v_c$. Therefore, the form factors that describe the wave function overlap between the initial and final mesons become independent of Dirac structure of weak current, and can only depend on a scalar quantity, $w \equiv v_b \cdot v_c$. Thus all form factors are related to a single universal function, $\xi(v_b \cdot v_c)$, the Isgur-Wise function, which contains all the low energy nonperturbative hadronic physics relevant for these decays. Moreover, $\xi(1) = 1$, because at the "zero recoil" point, $w = 1$, where the c quark is at rest in the b rest frame, the configuration of the brown muck does not change at all.

Using only Lorentz invariance, six form factors parameterize $B \to D^{(*)}\ell\bar{\nu}$ decay,

$$\begin{aligned}
\langle D(v')|V_\nu|B(v)\rangle &= \sqrt{m_B m_D}\, \Big[h_+\,(v+v')_\nu + h_-\,(v-v')_\nu\Big], \\
\langle D^*(v')|V_\nu|B(v)\rangle &= i\sqrt{m_B m_{D^*}}\, h_V\, \epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha} v'^\beta v^\gamma, \\
\langle D(v')|A_\nu|B(v)\rangle &= 0, \qquad\qquad (51)\\
\langle D^*(v')|A_\nu|B(v)\rangle &= \sqrt{m_B m_{D^*}}\, \Big[h_{A_1}\,(w+1)\epsilon^*_\nu - h_{A_2}\,(\epsilon^*\cdot v)v_\nu - h_{A_3}\,(\epsilon^*\cdot v)v'_\nu\Big],
\end{aligned}$$

where the h_i are functions of $w \equiv v\cdot v' = (m_B^2 + m_{D^{(*)}}^2 - q^2)/(2m_B m_{D^{(*)}})$. The currents relevant for semileptonic decay are $V_\nu = \bar{c}\gamma_\nu b$ and $A_\nu = \bar{c}\gamma_\nu\gamma_5 b$. In the $m_Q \to \infty$ limit,

$$h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w)\,, \qquad h_-(w) = h_{A_2}(w) = 0\,. \qquad (52)$$

There are corrections to these relations for finite $m_{c,b}$, suppressed by powers of α_s and $\Lambda_{\rm QCD}/m_{c,b}$. The former are calculable, while the latter can only be parameterized, and that is where model dependence enters.

The determination of $|V_{cb}|$ from exclusive $B \to D^{(*)}\ell\bar{\nu}$ decay uses an extrapolation of the measured decay rate to zero recoil, $w = 1$. The rates can be schematically written as

$$\frac{\mathrm{d}\Gamma(B \to D^{(*)}\ell\bar\nu)}{\mathrm{d}w} = (\text{known factors})\,|V_{cb}|^2 \begin{cases}(w^2-1)^{1/2}\,\mathcal{F}_*^2(w)\,, & \text{for } B \to D^*,\\ (w^2-1)^{3/2}\,\mathcal{F}^2(w)\,, & \text{for } B \to D.\end{cases} \tag{53}$$

Both $\mathcal{F}(w)$ and $\mathcal{F}_*(w)$ are equal to the Isgur-Wise function in the $m_Q \to \infty$ limit, and in particular $\mathcal{F}_{(*)}(1) = 1$, allowing for a model independent determination of $|V_{cb}|$. The corrections are again suppressed by powers of α_s and $\Lambda_{\mathrm{QCD}}/m_{c,b}$ and are of the form

$$\begin{aligned}\mathcal{F}_*(1) &= 1_{(\text{Isgur-Wise})} + c_A(\alpha_s) + \frac{0_{(\text{Luke})}}{m_{c,b}} + \frac{(\text{lattice or models})}{m_{c,b}^2} + \ldots\,,\\ \mathcal{F}(1) &= 1_{(\text{Isgur-Wise})} + c_V(\alpha_s) + \frac{(\text{lattice or models})}{m_{c,b}} + \ldots\,.\end{aligned} \tag{54}$$

The perturbative corrections, $c_A = -0.04$ and $c_V = 0.02$, have been computed to order α_s^2 [51], and the yet higher order corrections should be below the 1% level. The order $\Lambda_{\mathrm{QCD}}/m_Q$ correction to $\mathcal{F}_*(1)$ vanishes due to Luke's theorem.[52] The terms indicated by (lattice or models) in Eqs. (54) are only known using phenomenological models or quenched lattice QCD at present. This is why the determination of $|V_{cb}|$ from $B \to D^*\ell\bar\nu$ is theoretically more reliable than that from $B \to D\ell\bar\nu$, although both QCD sum rules[53] and quenched lattice QCD[54] suggest that the order $\Lambda_{\mathrm{QCD}}/m_{c,b}$ correction to $\mathcal{F}(1)$ is small (giving $\mathcal{F}(1) = 1.02 \pm 0.08$ and 1.06 ± 0.02, respectively). Due to the extra $w^2 - 1$ helicity suppression near zero recoil, $B \to D\ell\bar\nu$ is also harder experimentally than $B \to D^*\ell\bar\nu$. Reasonable estimates of $\mathcal{F}_*(1)$ are around

$$\mathcal{F}_*(1) = 0.91 \pm 0.04\,. \tag{55}$$

This value is unchanged for over five years,[4] and is supported by a recent lattice result.[55] The zero recoil limit of the $B \to D^*\ell\bar\nu$ rate is measured to be[26]

$$|V_{cb}|\,\mathcal{F}_*(1) = (38.3 \pm 1.0) \times 10^{-3}\,, \tag{56}$$

yielding $|V_{cb}| = (42.1 \pm 1.1_{\mathrm{exp}} \pm 1.9_{\mathrm{th}}) \times 10^{-3}$.

Another important theoretical input is the shape of $\mathcal{F}_{(*)}(w)$ used to fit the data. It is useful to expand about zero recoil and write $\mathcal{F}_{(*)}(w) = \mathcal{F}_{(*)}(1)\,[1 - \rho^2_{(*)}(w-1) + c_{(*)}(w-1)^2 + \ldots]$. Analyticity imposes stringent constraints between the slope, ρ^2, and curvature, c, at zero recoil,[56] which is used in the experimental fits to the data. Measuring the $B \to D\ell\bar\nu$ rate is also important, because computing $\mathcal{F}(1)$ on the lattice is not harder than $\mathcal{F}_*(1)$. Other cross-checks will come from ratios of the form factors in $B \to D^*\ell\bar\nu$, and comparing the shapes of the $B \to D^*$ and $B \to D$ spectra.[57] These can give additional constraints on ρ^2, which is important because the correlation between ρ^2 and the extracted value of $|V_{cb}|\,\mathcal{F}_*(1)$ is very large.

2.2.2 $B \to$ light form factors and SCET

In B decays to light mesons, there is a much more limited use of heavy quark symmetry, since it does not apply for the final state. One can still derive relations between the $B \to \rho\ell\bar{\nu}$, $K^*\ell^+\ell^-$, and $K^*\gamma$ form factors in the large q^2 region.[58] One can also relate the form factors that occur in B and D decays to one another. But the symmetry neither reduces the number of form factors, nor does it determine their normalization at any value of q^2. For example, it is possible to predict $B \to \rho\ell\bar{\nu}$ from the measured $D \to K^*\ell\bar{\nu}$ form factors, using the symmetries:

$$\begin{array}{ccccc} & \overline{B} & \xrightarrow{\bar{u}\Gamma b\, V_{ub}} & \rho\,\ell\bar{\nu} & \\ {\scriptstyle \text{flavor} \atop \scriptstyle SU(2)} & \updownarrow & & \updownarrow & {\scriptstyle \text{chiral} \atop \scriptstyle SU(3)} \\ & D & \xrightarrow{\bar{d}\Gamma c\, V_{cs}} & K^*\ell\bar{\nu} & \end{array} \tag{57}$$

The form factor relations hold at fixed value of $v \cdot v'$, that is, at the same energy of the light mesons in the heavy meson rest frame. The validity of these relations is also limited to order one values of $v \cdot v'$. (While maximal recoil in $B \to D^*$ and $B \to D$ decays are $v \cdot v' \simeq 1.5$ and 1.6, respectively, it is 3.5 in $B \to \rho$ and 18.9 in $B \to \pi$.) A limitation of this approach is that corrections to both heavy quark symmetry and chiral symmetry could be $\sim 20\%$ or more each. It may ultimately be possible to eliminate all first order symmetry breaking corrections[59] forming a “Grinstein type double ratio”[60] of the form factors that occur in the four decays $(B, D) \to (\rho, K^*)$, but this method will require very large data sets. The same region of phase space (large q^2 and modest light meson energy) is also the most accessible to lattice QCD calculations.

There have been important recent developments toward a better understanding of these form factors in the $q^2 \ll m_B^2$ region. It was proposed some time ago that in the heavy mass limit heavy-to-light semileptonic form factors become calculable in perturbative QCD.[61] There were several problems justifying such a proposal; for example, diagrams of the type in Fig. 11 can give contributions proportional to $1/x^2$ leading to singular integrals (x is the momentum fraction of one of the quarks). There have been many attempts to separate “soft” and “hard” contributions and understand how Sudakov effects might regulate the singularities.[62]

It was recently proposed[63] that the 7 form factors that parameterize matrix elements of all possible currents (V, A, S, P, T) in $B \to$ vector meson (ρ or K^*) transitions have extra symmetries and can be expressed in terms of two functions, $\xi_\perp(E)$ and $\xi_\parallel(E)$, in the limit where $m_b \to \infty$ and $E_{\rho,K^*} = \mathcal{O}(m_b)$. In the same limit, the 3 form factors that parameterize decays to pseudoscalars (π or K) are related to one function, $\xi_P(E)$. Loosely speaking, these relations were expected to arise because soft gluons cannot flip the helicity of the energetic light quark emerging from the weak decay.

A new effective field theory, the soft-collinear effective theory (SCET),[65–67] is being developed, that is a systematic framework to describe from first principles the interac-

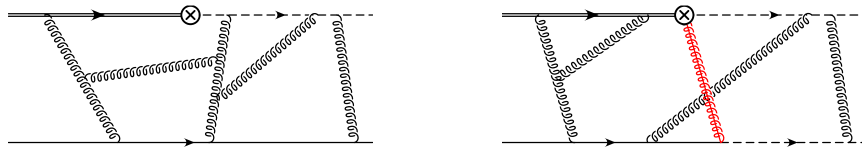


Fig. 11. Contributions to heavy-to-light form factors: "soft" nonfactorizable part (left), and "hard" factorizable part (right). Note that these pictures are somewhat misleading, as explained in the text. (From Ref. [64].)

tions of energetic but low invariant mass particles with soft quanta. The dynamics of a light quark moving along the z direction with large energy Q is simplest to describe decomposing its momentum in terms of the light-cone coordinates, $p = (p^-, p_\perp, p^+)$,

$$p^\mu = \bar{n}\cdot p\,\frac{n^\mu}{2} + p_\perp^\mu + n\cdot p\,\frac{\bar{n}^\mu}{2} \equiv p^-\,\frac{n^\mu}{2} + p_\perp^\mu + p^+\,\frac{\bar{n}^\mu}{2} \sim \Big[\mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \mathcal{O}(\lambda^2)\Big]\,Q\,, \tag{58}$$

where $n = (1,0,0,1)$ and $\bar{n} = (1,0,0,-1)$ are light-cone vectors ($n^2 = 0$), and $\lambda \sim \mathcal{O}(|p_\perp|/p^-)$ is a small parameter (please do not confuse it with the Wolfenstein parameter!). We have used that the on-shell condition imposes $p^+p^- \sim p_\perp^2 \sim \lambda^2\,Q^2$. In most applications $\lambda \sim \sqrt{\Lambda_{\rm QCD}/m_b}$ or $\Lambda_{\rm QCD}/m_b$. The goal is to separate contributions from the scales $p^2 \sim Q^2$, $Q\Lambda_{\rm QCD}$, and $\Lambda_{\rm QCD}^2$.

Similar to the field redefinition in HQET in Eq. (47), one can remove the large component of the momentum of a collinear quark by a filed redefinition[66]

$$\psi(x) = e^{-i\widetilde{p}\cdot x}\,\xi_n(x)\,, \tag{59}$$

where $\widetilde{p} = p^-\,n/2 + p_\perp$ contain the parts of the light quark momentum that can be parametrically larger than $\Lambda_{\rm QCD}$. An important complication compared to HQET is that $\widetilde{p}$ is not a fixed label on the collinear quark fields (in the sense that the four-velocity, v, is on heavy quarks), since emission of collinear gluons by a massless quark is not suppressed and changes $\widetilde{p}$. Therefore, one has to introduce separate collinear gluon fields in addition to collinear quarks and antiquarks. SCET gives an operator formulation of this complicated dynamics with well-defined power counting that simplifies all order proofs of factorization theorems, while previously such processes were analyzed only in terms of Feynman diagrams.

As far as heavy-to-light form factors are concerned, the relevant region of phase space is the small q^2 region, when m_M/E_M is small. The goal is to have a clean separation of contributions from momentum regions $p^2 \sim E_M^2$, $E_M\Lambda_{\rm QCD}$, and $\Lambda_{\rm QCD}^2$. There are two crucial questions when setting up such a framework. First, it has to be proven that such a separation is possible to all orders in the strong interaction. It was first shown at leading order in α_s that the infrared divergences can be absorbed into the soft form factors.[68] However, the relative size of the soft and hard contributions depend on assumptions about the tail of the pion wave function[68] or on the suppression of the soft part due to Sudakov effects.[69] SCET allows to prove factorization without such assumptions, to all orders in α_s and to leading order in $1/Q(\equiv 1/E_M)$.[70] A generic form factor can be split to two contributions $F(Q) = f^{\rm F}(Q) + f^{\rm NF}(Q)$, where the two terms arise from matrix elements of distinct operators between the same states. One can write[70]

$$\begin{aligned} f^{\rm F}(Q) &= \frac{f_B f_M}{Q^2} \int_0^1 {\rm d}z \int_0^1 {\rm d}x \int_0^\infty {\rm d}r_+\, T(z, Q, \mu_0) \\ &\qquad\qquad \times J(z, x, r_+, Q, \mu_0, \mu)\, \phi_M(x, \mu)\, \phi_B^+(r_+, \mu)\,, \\ f^{\rm NF}(Q) &= C_k(Q, \mu)\, \zeta_k^M(Q, \mu)\,. \end{aligned} \tag{60}$$

The hard coefficients, T, C_k, and J are process dependent; C_k and T can be calculated in an expansion in $\alpha_s(Q)$, while the so-called jet function, J, is dominated by momenta $p^2 \sim Q\Lambda_{\rm QCD}$ and starts at order $\alpha_s(\sqrt{Q\Lambda_{\rm QCD}})$. In Eq. (60) ϕ_M and ϕ_B^+ are nonperturbative distribution amplitudes for the final meson M and the initial B, on which both contributions depend. The nonfactorizable part depends on three soft form factors, ζ_k^M, which are universal nonperturbative functions. Only one occurs for decays to pseudoscalars, and two for decays to vector mesons, thus reproducing the heavy-to-light form factor relations.[63] The second question is to understand the power counting of the two contributions, including possible suppressions by α_s. Both terms in Eq. (60) scale as $(\Lambda_{\rm QCD}/Q)^{3/2}$. It is yet unknown whether the $f^{\rm NF}$ term might also have an $\alpha_s(\sqrt{Q\Lambda_{\rm QCD}})$ suppression,[70] similar to that present in J. Progress in theory is expected to answer this in the formal $m_b \gg \Lambda_{\rm QCD}$ limit, and testing the one relation between the three experimentally measurable $B \to \rho\ell\bar\nu$ form factors could tell us about the relative size of the two contributions for the physical b quark mass.

There are many possible applications. For example, one could use the $B \to K^*\gamma$ rate to constrain the $B \to \rho\ell\bar\nu$ and $B \to K^*\ell^+\ell^-$ form factors relevant for the determination of $|V_{ub}|$ and searches for new physics.[71] Some others are discussed in Sec. 2.4.2.

2.3 Inclusive semileptonic B decays

Sometimes, instead of identifying all particles in a decay, it is convenient to be ignorant about some details. For example, we might want to specify the energy of a charged lepton or a photon in the final state, or restrict the flavor of the final hadrons. These decays are inclusive in the sense that we sum over final states which can be produced by strong interactions, subject to a limited set of constraints determined by short distance perturbative physics. Typically we are interested in a quark-level transition, such as $b \to c\ell\bar\nu$, $b \to s\gamma$, etc., and we would like to extract the corresponding short distance parameters, $|V_{cb}|$, $C_7(m_b)$, etc., from the data. To do this, we need to be able to model independently relate the quark-level operators to the experimentally accessible observables.

2.3.1 The OPE, total rates, and $|V_{cb}|$

In the large m_b limit, when the energy released in the decay is large, there is a simple heuristic argument that the inclusive rate may be modeled simply by the decay of a free b quark. The argument is again based on a separation of time (or distance) scales. The b quark decay mediated by weak interaction takes place on a time scale that is much shorter than the time it takes the quarks in the final state to form physical hadronic states. Once the b quark has decayed on a time scale $t \ll \Lambda_{\rm QCD}^{-1}$, the probability that the final states will hadronize somehow is unity, and we need not know the (uncalculable) probabilities of hadronization into specific final states.

Let us consider inclusive semileptonic $b \to c$ decay, mediated by the operator

$$O_{\rm sl} = -\frac{4G_F}{\sqrt{2}}\, V_{cb}\, (J_{bc})^\alpha\, (J_{\ell\nu})_\alpha\,, \tag{61}$$

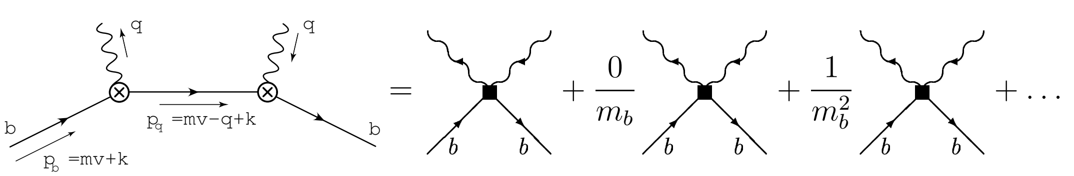
where $J_{bc}^\alpha = (\bar c\, \gamma^\alpha P_L\, b)$ and $J_{\ell\nu}^\beta = (\bar\ell\, \gamma^\beta P_L\, \nu)$. The decay rate is given by the square of the matrix element, integrated over phase space, and summed over final states,

$$\Gamma(B \to X_c \ell\bar\nu) \sim \sum_{X_c} \int {\rm d[PS]}\, \big|\langle X_c \ell\bar\nu | O_{\rm sl} | B\rangle\big|^2 . \tag{62}$$

Since the leptons have no strong interaction, it is convenient to factorize the phase space into $B \to X_c W^*$ and a perturbatively calculable leptonic part, $W^* \to \ell\bar\nu$. The nontrivial part is the hadronic tensor,

$$\begin{aligned} W^{\alpha\beta} &\sim \sum_{X_c} \delta^4(p_B - q - p_X)\, \big|\langle B| J_{bc}^{\alpha\dagger} |X_c\rangle\, \langle X_c| J_{bc}^\beta |B\rangle\big|^2 \\ &\sim {\rm Im} \int {\rm d}x\, e^{-iq\cdot x}\, \langle B|\, T\{J_{bc}^{\alpha\dagger}(x)\, J_{bc}^\beta(0)\}\, |B\rangle\,. \end{aligned} \tag{63}$$

where the second line is obtained using the optical theorem, and T denotes the time ordered product of the two operators. This is convenient, because it is this time ordered product that can be expanded in local operators in the $m_b \gg \Lambda_{\rm QCD}$ limit.[72] In this limit the time ordered product is dominated by short distances, $x \ll \Lambda_{\rm QCD}^{-1}$, and one can express the nonlocal hadronic tensor $W^{\alpha\beta}$ as a sum of local operators. Schematically,



(64)

This is analogous to a multipole expansion. At leading order the decay rate is determined by the b quark content of the initial state, while subleading effects are parameterized by matrix elements of operators with increasing number of derivatives that are sensitive to the distribution of chromomagnetic and chromoelectric fields.

At lowest order in $\Lambda_{\rm QCD}/m_b$ this operator product expansion (OPE) leads to operators of the form $\bar b\,\Gamma\, b$, where Γ is some (process-dependent) Dirac matrix. For $\Gamma=\gamma^\mu$ or $\gamma^\mu\gamma_5$ their matrix elements are known to all orders in $\Lambda_{\rm QCD}/m_b$

$$\begin{aligned}\langle B(p_B)|\,\bar b\,\gamma^\mu b\,|B(p_B)\rangle &= 2p_B^\mu = 2m_B\, v^\mu\,, \\ \langle B(p_B)|\,\bar b\,\gamma^\mu\gamma_5\, b\,|B(p_B)\rangle &= 0\,,\end{aligned} \tag{65}$$

because of conservation of the b quark number and parity invariance of strong interactions. The matrix elements for other Γ's can be related by heavy quark symmetry to these plus $\mathcal{O}(\Lambda_{\rm QCD}^2/m_b^2)$ terms. Thus the OPE justifies that inclusive B decay rates in the $m_b\to\infty$ limit are given by free b quark decay.

To compute subleading corrections, it is convenient to use HQET. There are no $\mathcal{O}(\Lambda_{\rm QCD}/m_b)$ corrections, because the B meson matrix element of any dimension-4 operator vanishes, $\langle B(v)|\,\bar h_v^{(b)} iD_\alpha\Gamma\, h_v^{(b)}|B(v)\rangle \,=\, 0$. The leading nonperturbative effects suppressed by $\Lambda_{\rm QCD}^2/m_b^2$ are parameterized by two HQET matrix elements,

$$\lambda_1 = \frac{1}{2m_B}\,\langle B|\,\bar h_v^{(b)}(iD)^2\, h_v^{(b)}\,|B\rangle\,, \qquad \lambda_2 = \frac{1}{6m_B}\,\langle B|\,\bar h_v^{(b)}\,\frac{g_s}{2}\,\sigma_{\mu\nu}\,G^{\mu\nu}\,h_v^{(b)}\,|B\rangle\,. \tag{66}$$

The B^*-B mass splitting determines $\lambda_2=(m_{B^*}^2-m_B^2)/4\simeq 0.12\,{\rm GeV}^2$, whereas the most promising way to determine λ_1 is from experimental data on inclusive decay distributions, as explained below. The result of the OPE can then be written schematically as

$${\rm d}\Gamma = \begin{pmatrix} b \text{ quark} \\ \text{decay}\end{pmatrix} \times \left\{1+\frac{0}{m_b}+\frac{f(\lambda_1,\lambda_2)}{m_B^2}+\ldots+\alpha_s(\ldots)+\alpha_s^2(\ldots)+\ldots\right\}. \tag{67}$$

At order $\Lambda_{\rm QCD}^3/m_b^3$, six new and largely unknown hadronic matrix elements enter, and usually naive dimensional analysis is used to estimate the uncertainties related to them. For most quantities of interest, the perturbation series are known including the α_s and $\alpha_s^2\beta_0$ terms, where $\beta_0=11-2n_f/3$ is the first coefficient of the QCD β-function (in many cases this term is expected to dominate the order α_s^2 corrections).

In which regions of phase space can the OPE be expected to converge? Near boundaries of the Dalitz plot the assumption that the energy release to the final hadronic state is large can be violated. It is useful to think of the OPE as an expansion in the residual momentum of the b quark, k, in the diagram on the left-hand side of Eq. (64). Expanding the propagator,

$$\frac{1}{(m_b v+k-q)^2-m_q^2} = \frac{1}{[(m_b v-q)^2-m_q^2]+[2k\cdot(m_b v-q)]+k^2}\,, \tag{68}$$

we see that for the expansion in powers of k to converge, the final state phase space can only be restricted in a manner to still allow hadronic final states X to contribute with

$$m_X^2-m_q^2 \gg E_X\Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2\,. \tag{69}$$

Before discussing the implications of this inequality, it has to be mentioned that the OPE implicitly relies on quark-hadron duality.[73] This is simply the notion that averaged over sufficiently many exclusive final states, hadronic quantities can be computed at the parton level. Its violations are believed to be small for fully inclusive semileptonic B decay rates (although this is not undisputed[74]), however, exactly how small is very hard to quantify. Comparing differential distributions discussed below appears to be the most promising way to constrain it experimentally.

The good news from Eq. (69) is that the OPE calculation of total rates should be under good control. The theoretical uncertainty is dominated by the uncertainty in a short distance b quark mass (whatever way it is defined) and in the perturbation series. Using the "upsilon expansion", the relation between the inclusive semileptonic rate and $|V_{cb}|$ is[75]

$$|V_{cb}| = (41.9 \pm 0.8_{(\rm pert)} \pm 0.5_{(m_b)} \pm 0.7_{(\lambda_1)}) \times 10^{-3} \left(\frac{\mathcal{B}(B \to X_c \ell \bar\nu)}{0.105} \, \frac{1.6\,\rm ps}{\tau_B} \right)^{1/2} . \quad (70)$$

The first error is from the uncertainty in the perturbation series, the second one from the b quark mass, $m_b^{1S} = 4.73 \pm 0.05\,\mathrm{GeV}$ (a very conservative range of m_b may be larger[76]), and the third one from $\lambda_1 = -0.25 \pm 0.25\,\mathrm{GeV}^2$. This result is in agreement with Ref. [77], where the central value is 40.8×10^{-3} (including a small, 1.007, electromagnetic radiative correction).

Progress in the determinations of m_b and λ_1 is likely to come from measurements of shape variables in inclusive B decays.[78] The idea is to look at decay distributions independent of CKM elements to learn about the hadronic parameters, that can in turn reduce the errors of the CKM measurements. Such observables are ratios of differently weighted integrals of decay distributions (sometimes called "moments"); specifically the charged lepton energy[79–82] and hadronic invariant mass[83,81] spectra in $B \to X_c \ell \bar\nu$ and the photon energy spectrum in $B \to X_s \gamma$.[84–86] Comparing these shape variables is also the most promising approach to constrain experimentally the accuracy of OPE, including the possible size of quark-hadron duality violation. The presently available measurements[87,88] do not seem to fit well together. It appears crucial to determine the $B \to D^{(*)} \ell \bar\nu$ branching ratios with higher precision, to model independently map out the hadronic invariant mass distribution in $B \to X_c \ell \bar\nu$ decay, and to try to measure the $B \to X_s \gamma$ spectrum to as low photon energies as possible. If the overall agreement improves, then this program may lead to an error in $|V_{cb}|$ at the $\sim 2\%$ level.

The bad news from Eq. (69) is that in certain restricted regions of phase space the OPE breaks down. This is a problem, for example, for the determination of $|V_{ub}|$ from $B \to X_u \ell \bar\nu$, because severe cuts are required to eliminate ~ 100 times larger $b \to c$ background. Similarly, in $B \to X_s \gamma$, the rate can only be measured for energetic photons that populate a modest region of phase space, $E_\gamma^{\rm max} - E_\gamma^{\rm min} < 1\,\mathrm{GeV}$. Some of the new theoretical problems that enter in such situations are discussed next.

2.3.2 $B \to X_u \ell \bar\nu$ spectra and $|V_{ub}|$

If it were not for the huge $B \to X_c \ell \bar\nu$ background, measuring $|V_{ub}|$ would be as "easy" as $|V_{cb}|$. The total $B \to X_u \ell \bar\nu$ rate can be predicted in the OPE with small uncertainty,[75]

$$|V_{ub}| = (3.04 \pm 0.06_{(\rm pert)} \pm 0.08_{(m_b)}) \times 10^{-3} \left(\frac{\mathcal{B}(B \to X_u \ell \bar\nu)}{0.001} \frac{1.6\,\rm ps}{\tau_B} \right)^{1/2}, \qquad (71)$$

where the errors are as discussed after Eq. (70). If this fully inclusive rate is measured without significant cuts on the phase space, then $|V_{ub}|$ may be determined with less than 5% theoretical error.

When kinematic cuts are used to distinguish the $b \to u$ signal from the $b \to c$ background, the behavior of the OPE can become significantly worse. As indicated by Eq. (69), there are three qualitatively different regions of phase space, depending on how the invariant mass and energy of the hadronic final state (in the B rest frame) is restricted:

(i) $m_X^2 \gg E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: the OPE converges, and the first few terms are expected to give reliable result. This is the case for the $B \to X_c \ell \bar\nu$ width relevant for measuring $|V_{cb}|$.

(ii) $m_X^2 \sim E_X \Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$: an infinite set of equally important terms in the OPE must be resummed. The OPE becomes a twist expansion and nonperturbative input is needed.

(iii) $m_X \sim \Lambda_{\rm QCD}$: the final state is dominated by resonances, and it is not known how to compute any inclusive quantity reliably.

The charm background can be removed by several different kinematic cuts:

1. $E_\ell > (m_B^2 - m_D^2)/(2m_B)$: the lepton endpoint region that was used to first observe $b \to u$ decay;
2. $m_X < m_D$: the small hadronic invariant mass region;[89–92]
3. $E_X < m_D$: the small hadronic energy region;[93]
4. $q^2 \equiv (p_\ell + p_\nu)^2 > (m_B - m_D)^2$: the large dilepton invariant mass region.[94]

These contain roughly 10%, 80%, 30%, and 20% of the rate, respectively. Measuring any other variable than E_ℓ requires the reconstruction of the neutrino, which is challenging experimentally. Combinations of cuts have also been proposed, q^2 with m_X [95], q^2 with E_ℓ [96], or m_X with E_X [97].

The problem is that both phase space regions 1. and 2. belong to the regime (ii), because these cuts impose $m_X \lesssim m_D$ and $E_X \lesssim m_B$, and numerically $\Lambda_{\rm QCD}\, m_B \sim m_D^2$. The region $m_X < m_D$ is better than $E_\ell > (m_B^2 - m_D^2)/(2m_B)$ inasmuch as the expected rate is larger, and the inclusive description is expected to hold better. But nonperturbative input is needed in both cases, formally at the $\mathcal{O}(1)$ level, which is why the model dependence increases rapidly if the m_X cut is lowered below m_D.[90] These regions of the Dalitz plot are shown in Fig. 12.

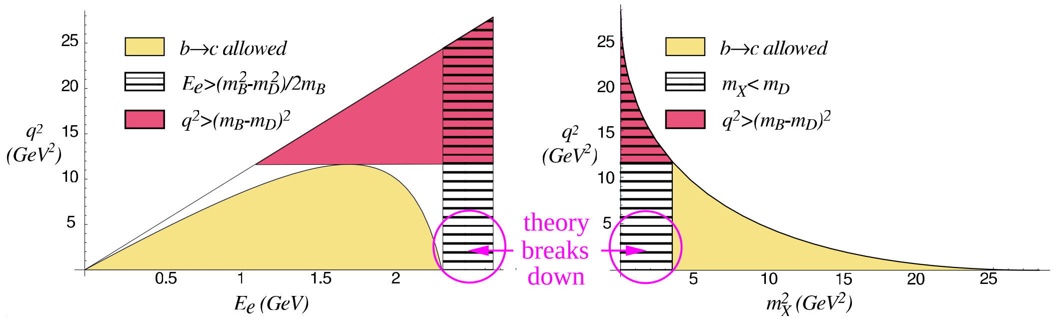


Fig. 12. Dalitz plots for $B \to X\ell\bar{\nu}$ in terms of E_ℓ and q^2 (left), and m_X^2 and q^2 (right).

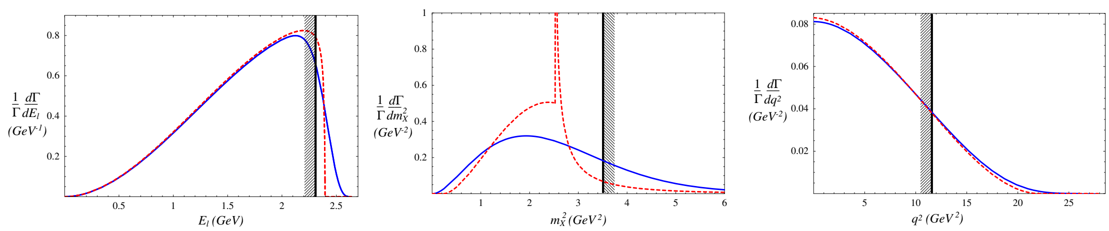


Fig. 13. $B \to X_u\ell\bar{\nu}$ spectra — E_ℓ (left), m_X^2 (middle), and q^2 (right) — as given by b quark decay including $\mathcal{O}(\alpha_s)$ terms (dashed curves), and including the Fermi motion model (solid curves).

The large E_ℓ and small m_X regions are determined by the b quark light-cone distribution function that describes the Fermi motion inside the B meson (sometimes called the shape function). Its effect on the spectra are illustrated in Fig. 13, where we also show the q^2 spectrum unaffected by it. This nonperturbative function is universal at leading order in $\Lambda_{\rm QCD}/m_b$, and is related to the $B \to X_s\gamma$ photon spectrum.[98] These relations have been extended to the resummed next-to-leading order corrections,[99] and to include effects of operators other than O_7 contributing to $B \to X_s\gamma$.[100] Weighted integrals of the $B \to X_s\gamma$ photon spectrum are equal to the $B \to X_u\ell\bar{\nu}$ rate in the large E_ℓ or small m_X regions. Recently CLEO[101] used the $B \to X_s\gamma$ photon spectrum as an input to determine $|V_{ub}| = (4.08 \pm 0.63) \times 10^{-3}$ from the lepton endpoint region.

The dominant theoretical uncertainty in this determinations of $|V_{ub}|$ are from subleading twist contributions, which are not related to $B \to X_s\gamma$.[102] The $B \to X_u\ell\bar{\nu}$ lepton spectrum, including dimension-5 operators and neglecting perturbative corrections, is given by[72]

$$\frac{{\rm d}\Gamma}{{\rm d}y} = \frac{G_F^2\, m_b^5\, |V_{ub}|^2}{192\,\pi^3} \bigg\{ \bigg[y^2(3-2y) + \frac{5\lambda_1}{3m_b^2}\, y^3 + \frac{\lambda_2}{m_b^2}\, y^2(6+5y) \bigg]\, 2\theta(1-y) \\ - \bigg[\frac{\lambda_1}{6m_b^2} + \frac{11\lambda_2}{2m_b^2} \bigg]\, 2\delta(1-y) - \frac{\lambda_1}{6m_b^2}\, 2\delta'(1-y) + \ldots \bigg\}. \quad (72)$$

The behavior near $y = 1$ is determined by the leading order structure function, which contains the terms $2[\theta(1-y) - \lambda_1/(6m_b^2)\,\delta'(1-y) + \ldots]$. The derivative of the same combination occurs in the $B \to X_s\gamma$ photon spectrum,[103] given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \frac{G_F^2\, m_b^5\, |V_{tb}V_{ts}^*|^2\, \alpha\, C_7^2}{32\,\pi^4} \bigg[\bigg(1 + \frac{\lambda_1 - 9\lambda_2}{2m_b^2}\bigg)\, \delta(1-x) - \frac{\lambda_1 + 3\lambda_2}{2m_b^2}\, \delta'(1-x) - \frac{\lambda_1}{6m_b^2}\, \delta''(1-x) + \ldots\bigg], \tag{73}$$

At subleading order, proportional to $\delta(1-y)$ in Eq. (72) and to $\delta'(1-x)$ in Eq. (73), the terms involving λ_2 differ significantly, with a coefficient $11/2$ in Eq. (72) and $3/2$ in Eq. (73). Because of the 11/2 factor, the $\lambda_2\,\delta(1-y)$ term is important in the lepton endpoint region.[102,104,105] There is also a significant uncertainty at order $\Lambda_{\rm QCD}^2/m_b^2$ from weak annihilation.[106,102] Moreover, if the lepton endpoint region is found to be dominated by the π and ρ exclusive channels, then the applicability of the inclusive description may be questioned.

In contrast to the above, in the $q^2 > (m_B - m_D)^2$ region the first few terms in the OPE determine the rate.[94] This cut implies $E_X \lesssim m_D$ and $m_X \lesssim m_D$, and so the $m_X^2 \gg E_X\Lambda_{\rm QCD} \gg \Lambda_{\rm QCD}^2$ criterion of regime (i) is satisfied. This relies, however, on $m_c \gg \Lambda_{\rm QCD}$, and so the OPE is effectively an expansion in $\Lambda_{\rm QCD}/m_c$.[107] The largest uncertainties come from order $\Lambda_{\rm QCD}^3/m_{c,b}^3$ nonperturbative corrections, the b quark mass, and the perturbation series. Weak annihilation (WA) suppressed by $\Lambda_{\rm QCD}^3/m_b^3$ is important, because it enters the rate as $\delta(q^2 - m_b^2)$.[106] Its magnitude is hard to estimate, because it is proportional to the difference of two matrix elements, which are equal in the factorization limit. Assuming a 10% violation of factorization, WA could be $\sim 2\%$ of the $B \to X_u\ell\bar\nu$ rate, and, in turn, $\sim 10\%$ of the rate in the $q^2 > (m_B - m_D)^2$ region. The uncertainty of this estimate is large. Since this contribution is also proportional to $\delta(E_\ell - m_b/2)$, it is even more important for the lepton endpoint region. Experimentally, WA can be constrained by comparing $|V_{ub}|$ measured from B^0 and $B^\pm$ decays, and by comparing the D^0 and D_s semileptonic widths.[106]

Combining the q^2 and m_X cuts can significantly reduce the theoretical uncertainties.[95] The right-hand side of Fig. 12 shows that the q^2 cut can be lowered below $(m_B - m_D)^2$ by imposing an additional cut on m_X. This changes the expansion parameter from $\Lambda_{\rm QCD}/m_c$ to $m_b\Lambda_{\rm QCD}/(m_b^2 - q_{\rm cut}^2)$, resulting in a significant decrease of the uncertainties from both the perturbation series and from the nonperturbative corrections. At the same time the uncertainty from the b quark light-cone distribution function only turns on slowly. Some representative results are give in Table 3, showing that it may be possible to determine $|V_{ub}|$ with a theoretical error at the $\sim 5\%$ level using up to $\sim 45\%$ of the semileptonic decays.

Cuts on q^2 and m_X	Fraction of events	Error of $\lvert V_{ub}\rvert$ $\delta m_b = 80/30\,\mathrm{MeV}$
$6\,\mathrm{GeV}^2,\ m_D$	46%	$8\%/5\%$
$8\,\mathrm{GeV}^2,\ 1.7\,\mathrm{GeV}$	33%	$9\%/6\%$
$(m_B - m_D)^2,\ m_D$	17%	$15\%/12\%$

Table 3. $|V_{ub}|$ from combined cuts on q^2 and m_X (from Ref. [95]).

2.4 Some additional topics

This section contains short discussions of three topics that there was no time to cover during the lectures, but were included in the printed slides. Skipping this section will not affect the understanding of the rest of this writeup.

2.4.1 B decays to excited D mesons

Heavy quark symmetry implies that in the $m_Q \to \infty$ limit, matrix elements of the weak currents between a B meson and an excited charmed meson vanish at zero recoil. However, in some cases at order $\Lambda_{\rm QCD}/m_Q$ these matrix elements are nonzero and calculable.[108] Since most of the phase space is near zero recoil, $\Lambda_{\rm QCD}/m_Q$ corrections can be very important.

In the heavy quark limit, for each doublet of excited D mesons, all semileptonic decay form factors are related to a single Isgur-Wise function.[109] At $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ many new functions occur. In $B \to (D_1, D_2^*)\ell\bar\nu$ there are 8 subleading Isgur-Wise functions (neglecting time ordered products with subleading terms in the Lagrangian, which are expected to be small or can be absorbed), but only 2 of them are independent.[108] Moreover, in $B \to$ orbitally excited D decays, the zero recoil matrix element at $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ is given by mass splittings and the $m_Q \to \infty$ Isgur-Wise function. For example, in $B \to D_1\ell\bar\nu$ decay,[108]

$$f_{V_1}(1) = -\frac{4}{\sqrt{6}\,m_c}\,(\bar\Lambda' - \bar\Lambda)\,\tau(1)\,. \tag{74}$$

Here f_{V_1} is the form factor defined by

$$\langle D_1(v',\epsilon)|V^\mu|B(v)\rangle = \sqrt{m_{D_1} m_B}\,\Big[f_{V_1}\epsilon^{*\mu} + (f_{V_2}v^\mu + f_{V_3}v'^\mu)(\epsilon^*\cdot v)\Big]\,, \tag{75}$$

which determines the rate at zero recoil, similar to h_{A_1} in $B \to D^*$ decay defined in Eq. (51). Here τ denotes the leading order Isgur-Wise function, and $\bar\Lambda'$ is the $m_{D_1} - m_c$ mass splitting in the heavy quark limit ($\bar\Lambda' - \bar\Lambda \equiv \Delta_1$ in Fig. 9). Using Eq. (74), the decay rate can be expanded simultaneously in powers of $\Lambda_{\rm QCD}/m_Q$ and $w-1$ schematically as

$$
\begin{aligned}
\frac{{\rm d}\Gamma(B\to D_1\ell\bar\nu)}{{\rm d}w} \propto \sqrt{w^2-1}\,[\tau(1)]^2 \Big\{ & 0 + 0\,(w-1) + (\ldots)(w-1)^2 + \ldots \\
& + \frac{\Lambda_{\rm QCD}}{m_Q}\, \Big[0 + (\text{almost calculable})(w-1) + \ldots \Big] \\
& + \frac{\Lambda_{\rm QCD}^2}{m_Q^2}\, \Big[(\text{calculable}) + \ldots \Big] + \ldots \Big\}
\end{aligned}
\tag{76}
$$

The zeros and the calculable terms are model independent predictions of HQET, while the "almost calculable" term has a calculable part that is expected to be dominant.

There are many experimentally testable implications. One of the least model dependent is the prediction for

$$
R \equiv \frac{\mathcal{B}(B\to D_2^*\ell\bar\nu)}{\mathcal{B}(B\to D_1\ell\bar\nu)}\,,
\tag{77}
$$

because the leading order Isgur-Wise function drops out to a good approximation. This ratio is around 1.6 in the infinite mass limit, and it was predicted to be reduced to about $0.4-0.7$,[108] because $\Lambda_{\rm QCD}/m_c$ corrections enhance the $B\to D_1$ rate significantly but hardly affect $B\to D_2^*$. The present world average is about 0.4 ± 0.15.

To compare the $B\to(D_1,D_2^*)$ rates with (D_0^*,D_1^*), we need to know the leading Isgur-Wise functions. Quark models and QCD sum rules predict that the Isgur-Wise function for the broad (D_0^*,D_1^*) doublet is not larger than for the narrow (D_1,D_2^*) doublet.[110] These arguments make the large $B\to(D_0^*,D_1^*)\ell\bar\nu$ rates puzzling.

Another way the theory of these decays can be tested is via nonleptonic decays. Factorization in $B\to D^{**}\pi$ is expected to work as well as in $B\to D^{(*)}\pi$ (see Sec. 3.3),

$$
\Gamma_\pi = \frac{3\pi^2\,|V_{ud}|^2\,C^2 f_\pi^2}{m_B^2\,r} \times \left(\frac{{\rm d}\Gamma_{\rm sl}}{{\rm d}w}\right)_{w_{\rm max}}\,,
\tag{78}
$$

where $r=m_{D^{**}}/m_B$, $f_\pi\simeq 131\,$MeV, $w_{\rm max}=(1+r^2)/(2r)\simeq1.3$ in these decays, and $C\,|V_{ud}|\simeq1$. (As we will see in Sec. 3.3.1, this test would be more reliable in B^0 decay, however that is harder to measure experimentally.) An interesting ratio with little sensitivity to the leading order Isgur-Wise function was recently measured with good precision[111]

$$
R_\pi \equiv \frac{\mathcal{B}(B^-\to D_2^{*0}\pi^-)}{\mathcal{B}(B^-\to D_1^0\pi^-)} = 0.89\pm0.14\,,
\tag{79}
$$

whereas the CLEO result was 1.8 ± 0.9.[112] Figure 14 shows that R_π is very sensitive to the subleading $\mathcal{O}(\Lambda_{\rm QCD}/m_Q)$ Isgur-Wise functions, $\hat\tau_1$ and $\hat\tau_2$. Assuming that they are below $500\,$MeV (which is not an unusually large value by any means), the theory

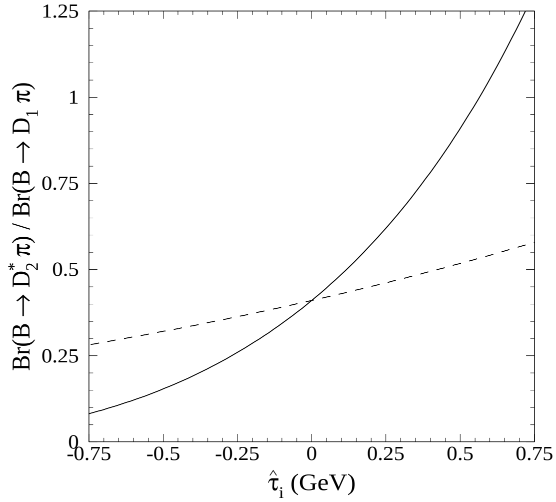


Fig. 14. Factorization prediction for R_π defined in Eq. (79) as a function of $\hat\tau_1$ for $\hat\tau_2 = 0$ (solid curve), and as a function of $\hat\tau_2$ for $\hat\tau_1 = 0$ (dashed curve). (From Ref. [108].)

predicts $R_\pi < 1$. Neglecting $\Lambda_{\rm QCD}/m_Q$ corrections,[113] the prediction is $R_\pi \sim 0.35$, as also seen from Fig. 14. We learn that the BELLE result in Eq. (79) agrees well with theory, which is a success of HQET in a regime with large sensitivity to $\Lambda_{\rm QCD}/m_Q$ effects. It constrains the subleading Isgur-Wise functions, which has useful implications for the analysis of $B \to D_1 \ell\bar\nu$ and $D_2^* \ell\bar\nu$ decays.

Sorting out these semileptonic and nonleptonic decays to excited D's will provide important tests of HQET, factorization, and will also impact the determinations of $|V_{cb}|$.

2.4.2 Exclusive rare decays

Exclusive rare decays are interesting for a large variety of reasons. As any flavor-changing neutral current process, they are sensitive probes of new physics, and within the SM they are sensitive to $|V_{td}|$ and $|V_{ts}|$. For example, $B \to K^{(*)}\ell^+\ell^-$ or $B \to X\,\ell^+\ell^-$ are sensitive to SUSY, enhanced bsZ penguins, right handed couplings, etc.

Exclusive rare decays are experimentally easier to measure than inclusive decays, but a clean theoretical interpretation requires model independent knowledge of the corresponding form factors. (However, certain CP asymmetries are independent of them.) It was originally observed that there is an observable, the forward-backward asymmetry in $B \to K^*\ell^+\ell^-$, A_{FB}, that vanishes at a value of the dilepton invariant mass, q^2, independent of form factor models[114] (near $q_0^2 = 4\,{\rm GeV}^2$ in the SM, see Fig. 15). This was shown to follow from the large energy limit,[63,66] as far as the soft contributions to the form factors are concerned. One finds the following implicit equation for q_0^2

$$C_9(q_0^2) = -C_7\,\frac{2m_B\,m_b}{q_0^2}\left[1 + \mathcal{O}\bigg(\text{“}\alpha_s\text{”}, \frac{\Lambda_{\rm QCD}}{m_b}\bigg)\right]. \tag{80}$$

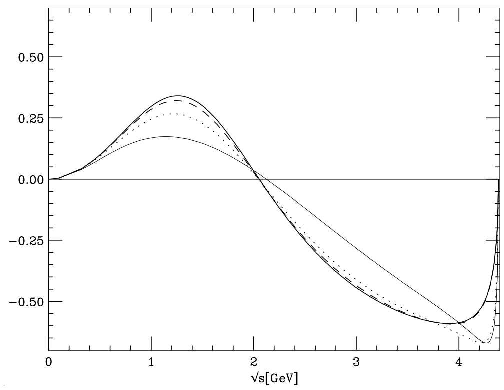


Fig. 15. Forward-backward asymmetry in $B \to K^* \ell^+ \ell^-$ decay in different form factor models ($s \equiv q^2$). (From Ref. [114].)

The quotation marks around the α_s corrections indicate that it is actually not known yet whether these are formally suppressed compared to the "leading" terms. The order α_s terms have been calculated,[68,115] but reliable estimates of the $\Lambda_{\rm QCD}/E_{K^*}$ terms are not available yet. It is hoped that with future theoretical developments the vanishing of A_{FB} will allow to search for new physics ; C_7 is known from $B \to X_s\gamma$, so the zero of A_{FB} determines C_9, which is sensitive to new physics ($C_{7,9}$ are the effective Wilson coefficients often denoted by $C_{7,9}^{\rm eff}$, and C_9 has a mild q^2-dependence).

There has also been considerable progress refining predictions for $B \to K^*\gamma$ and $\rho\gamma$. The calculations of $\mathcal{O}(\alpha_s)$ corrections show a strong enhancement ($\sim 80\%$) of the $B \to K^*\gamma$ rate.[115,116] The counting of α_s factors is again not firmly established yet.

The form factors also enter the prediction for the isospin splitting. These are power suppressed corrections, but were claimed to be calculable with some assumptions.[117] The prediction,

$$\Delta_{0-} = \frac{\Gamma(\overline{B}^0 \to \overline{K}^{*0}\gamma) - \Gamma(B^- \to K^{*-}\gamma)}{\Gamma(\overline{B}^0 \to \overline{K}^{*0}\gamma) + \Gamma(B^- \to K^{*-}\gamma)} = \frac{0.3}{T_1^{B\to K^*}} \times \left(0.08^{+0.02}_{-0.03}\right), \tag{81}$$

is to be compared with the present world average, 0.02 ± 0.07.

Testing these predictions is important in their own rights, and may also help to understand some assumptions entering factorization in charmless nonleptonic B decay.

2.4.3 Inclusive rare decays

Rare B decays are sensitive probes of new physics. There are many interesting modes sensitive to different extensions of the Standard Model. For example, $B \to X_s\gamma$ provides the best bound on the charged Higgs mass in type-II two Higgs doublet model, and also constrains the parameter space of SUSY models. Other rare decays such as $B \to X\ell^+\ell^-$ are sensitive through the bsZ effective coupling to SUSY and left-right

Decay mode	Approximate SM rate	Present status
$B \to X_s\gamma$	3.6×10^{-4}	$(3.4 \pm 0.4)\times 10^{-4}$
$B \to X_s\nu\bar\nu$	4×10^{-5}	$< 7.7 \times 10^{-4}$
$B \to \tau\nu$	4×10^{-5}	$< 5.7 \times 10^{-4}$
$B \to X_s\ell^+\ell^-$	5×10^{-6}	$(6 \pm 2) \times 10^{-6}$
$B_s \to \tau^+\tau^-$	1×10^{-6}	
$B \to X_s\tau^+\tau^-$	5×10^{-7}	
$B \to \mu\nu$	2×10^{-7}	$< 6.5 \times 10^{-6}$
$B_s \to \mu^+\mu^-$	4×10^{-9}	$< 2 \times 10^{-6}$
$B \to \mu^+\mu^-$	1×10^{-10}	$< 2.8 \times 10^{-7}$

Table 4. Some interesting rare decays, their SM rates, and present status.

symmetric models. $B \to X\nu\bar\nu$ can probe models containing unconstrained couplings between three 3rd generation fermions.[118]

We learned in the last year that the CKM contributions to rare decays are probably the dominant ones, as they are for CP violation in $B \to \psi K_S$. This is supported by the measurement of $\mathcal{B}(B \to X_s\gamma)$ which agrees with the SM at the 15% level[88,119]; the measurements of $B \to X_s\ell^+\ell^-$ and $B \to K\ell^+\ell^-$, which are in the ballpark of the SM expectation[120,121]; and the non-observation of direct CP violation in $b \to s\gamma$, $A_{CP}(B \to X_s\gamma) = -0.08 \pm 0.11$[122] and $A_{CP}(B \to K^*\gamma) = -0.02 \pm 0.05$,[123] which are expected to be tiny in the SM. These results make it unlikely that new physics yields order-of-magnitude enhancement of any rare decay. It is more likely that only a broad set of precision measurements will be able to find signals of new physics.

At present, inclusive rare decays are theoretically cleaner than the exclusive ones, since they are calculable in an OPE and precise multi-loop results exist (see Ref. [124] for a recent review). Table 4 summarizes some of the most interesting modes. The $b \to d$ rates are expected to be about a factor of $|V_{td}/V_{ts}|^2 \sim \lambda^2$ smaller than the corresponding $b \to s$ modes shown. As a guesstimate, in $b \to q\, l_1 l_2$ decays one expects $10 - 20\%$ K^*/ρ and $5 - 10\%$ K/π.

A source of worry (at least, to me) is the long distance contribution, $B \to \psi X_s$ followed by $\psi \to \ell^+\ell^-$, which gives a combined branching ratio $\mathcal{B}(B \to X_s\ell^+\ell^-) \approx (4 \times 10^{-3}) \times (6 \times 10^{-2}) \approx 2 \times 10^{-4}$. This is about 30 times the short distance contribution. Averaged over a large region of invariant masses (and $0 < q^2 < m_B^2$ should be large enough), the $c\bar{c}$ loop is expected to be dual to $\psi + \psi' + \ldots$. This is what happens in $e^+e^- \to$ hadrons, in τ decay, etc., but apparently not here. Is it then consistent to "cut out" the ψ and ψ' regions and then compare the data with the short distance calculation? Maybe yes, but our present understanding is not satisfactory.

2.5 Summary

- $|V_{cb}|$ is known at the $\sim 5\%$ level; error may become half of this in the next few years using both inclusive and exclusive determinations (latter will rely on lattice).
- Situation for $|V_{ub}|$ may become similar to present $|V_{cb}|$; for precise inclusive determination the neutrino reconstruction seems crucial (the exclusive will use lattice).
- For $|V_{cb}|$ and $|V_{ub}|$ important to pursue both inclusive and exclusive measurements.
- Progress in understanding heavy-to-light form factors in $q^2 \ll m_B^2$ region: $B \to \rho\ell\bar{\nu}$, $K^{(*)}\gamma$, and $K^{(*)}\ell^+\ell^-$ below the $\psi \Rightarrow$ increase sensitivity to new physics. Related to certain questions in factorization in charmless decays.

3 Future Clean CP Measurements, Nonleptonic B Decays, Conclusions

This last lecture discusses several topics which will play important roles in the future of B physics. First, the complications of a clean determination of the CKM angle α from $B \to \pi\pi$ decays, and how those might be circumvented. Then we discuss some future clean CP measurements, such as $B_s \to D_s K$ and $B \to DK$. Although some of these measurements are only doable at a super-B-factory and/or LHCb/BTeV, their theoretical cleanliness makes them important. The second half of the lecture deals with factorization in $B \to D^{(*)}X$ type decays and its tests, followed by the different approaches to factorization in charmless decays and some possible applications.

Effective Hamiltonians Nonleptonic B decays mediated by $\Delta B = -\Delta C = \pm 1$ transitions are the simplest hadronic decays, described by the effective Hamiltonian

$$H = \frac{4G_F}{\sqrt{2}}\, V_{cb} V_{uq}^* \sum_{i=1}^{2} C_i(\mu)\, O_i(\mu) + \text{h.c.}\,, \qquad (82)$$

where $q = s$ or d, and

$$O_1(\mu) = (\bar{q}_L^\alpha \gamma_\mu u_L^\beta)\, (\bar{c}_L^\beta \gamma^\mu b_L^\alpha)\,, \qquad O_2(\mu) = (\bar{q}_L^\alpha \gamma_\mu u_L^\alpha)\, (\bar{c}_L^\beta \gamma^\mu b_L^\beta)\,. \qquad (83)$$

Here α and β are color indices. The $\Delta B = \Delta C = \pm 1$ Hamiltonian is related to Eqs. (82)–(83) by the trivial $c \leftrightarrow u$ interchange.

Decays with $\Delta B = \pm 1$ and $\Delta C = 0$ are more complicated,

$$H = \frac{4G_F}{\sqrt{2}} \sum_{j=u,c} V_{jb} V_{jq}^* \sum_i C_i(\mu)\, O_i(\mu) + \text{h.c.}\,, \tag{84}$$

The C_i are calculable Wilson coefficients, known to high precision. To write Eq. (84), the unitarity relation $V_{tb}V_{tq}^* = -V_{cb}V_{cq}^* - V_{ub}V_{uq}^*$ is used to rewrite the CKM elements that occur in penguin diagrams with intermediate top quark in terms of the CKM elements that occur in tree diagrams. The operator basis is conventionally chosen as

$$\begin{aligned}
&O_1^j = (\bar q_L^\alpha \gamma_\mu j_L^\beta)\,(\bar j_L^\beta \gamma^\mu b_L^\alpha)\,, && O_2^j = (\bar q_L^\alpha \gamma_\mu j_L^\alpha)\,(\bar j_L^\beta \gamma^\mu b_L^\beta)\,, \\
&O_3 = \bar q_L^\alpha \gamma_\mu b_L^\alpha \sum_{q'} \bar q_L'^\beta \gamma^\mu q_L'^\beta\,, && O_4 = \bar q_L^\alpha \gamma_\mu b_L^\beta \sum_{q'} \bar q_L'^\beta \gamma^\mu q_L'^\alpha\,, \\
&O_5 = \bar q_L^\alpha \gamma_\mu b_L^\alpha \sum_{q'} \bar q_R'^\beta \gamma^\mu q_R'^\beta\,, && O_6 = \bar q_L^\alpha \gamma_\mu b_L^\beta \sum_{q'} \bar q_R'^\beta \gamma^\mu q_R'^\alpha\,, \\
&O_8 = -\frac{g}{16\pi^2}\, m_b\, \bar q_L \sigma^{\mu\nu} G_{\mu\nu} b_R\,,
\end{aligned} \tag{85}$$

where $j = c$ or u, and the sums run over $q' = \{u, d, s, c, b\}$. In O_8, $G_{\mu\nu}$ is the chromomagnetic field strength tensor. Usually O_1 and O_2 are called current-current operators, $O_3 - O_6$ are four-quark penguin operators, and O_8 is the chromomagnetic penguin operator. These operators arise at lowest order in the electroweak interaction, i.e., diagrams involving a single W boson and QCD corrections to it. In some cases, especially when isospin breaking plays a role, one also needs to consider penguin diagrams which are second order in $\alpha_{\rm ew}$. They give rise to the electroweak penguin operators,

$$\begin{aligned}
O_7 &= -\frac{e}{16\pi^2}\, m_b\, \bar q_L^\alpha\, \sigma^{\mu\nu} F_{\mu\nu}\, b_R^\alpha\,, \\
O_7^{\rm ew} &= \frac{3}{2}\, \bar q_L^\alpha \gamma_\mu b_L^\alpha \sum_{q'} e_{q'}\, \bar q_R'^\beta \gamma^\mu q_R'^\beta\,, \qquad O_8^{\rm ew} = \frac{3}{2}\, \bar q_L^\alpha \gamma_\mu b_L^\beta \sum_{q'} e_{q'}\, \bar q_R'^\beta \gamma^\mu q_R'^\alpha\,, \\
O_9^{\rm ew} &= \frac{3}{2}\, \bar q_L^\alpha \gamma_\mu b_L^\alpha \sum_{q'} e_{q'}\, \bar q_L'^\beta \gamma^\mu q_L'^\beta\,, \qquad O_{10}^{\rm ew} = \frac{3}{2}\, \bar q_L^\alpha \gamma_\mu b_L^\beta \sum_{q'} e_{q'}\, \bar q_L'^\beta \gamma^\mu q_L'^\alpha\,.
\end{aligned} \tag{86}$$

Here $F^{\mu\nu}$ is the electromagnetic field strength tensor, and $e_{q'}$ denotes the electric charge of the quark q'.

Sometimes the contributions to decay amplitudes are classified by the appearance of Feynman diagrams with propagating top quarks, W and Z bosons, and people talk about tree (T), color-suppressed tree (C), penguin (P), and weak annihilation or W-exchange (W) contributions. While this may be convenient in some cases, the resulting arguments can be misleading. The separation between these contributions is usually ambiguous, as the "tree" and "penguin" operators mix under the renormalization group.

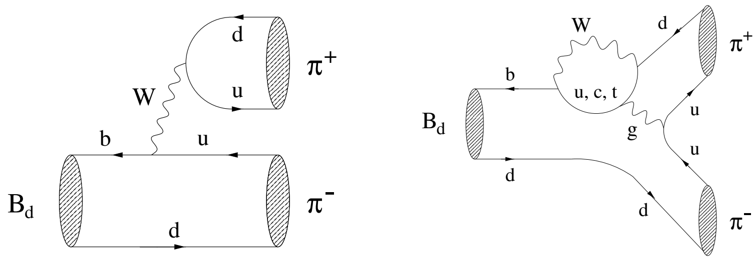


Fig. 16. "Tree" (left) and "Penguin" (right) contributions to $B \to \pi\pi$ (from Ref. [40]).

At the scale m_b the physics relevant for weak decays is described by the operators in Eqs. (83), (85), and (86), and their Wilson coefficients, and there are no propagating heavy particles. Usually one calls the O_1 and O_2 contributions (plus possibly a part of $O_3 - O_6$ and O_8) "tree", while $O_3 - O_6$ and O_8 (plus possibly a part of $O_1 - O_2$) "penguin". Below we will try to state clearly what is meant in each case.

3.1 $B \to \pi\pi$ — beware of penguins

We saw in Sec. 1.6.2 that the CP asymmetry in $B \to \psi K_S$ gives a theoretically very clean determination of $\sin 2\beta$, because the amplitude is dominated by contributions with a single weak phase. Similar to that case, there are tree and penguin contributions to the $B \to \pi^+\pi^-$ amplitude as well, as shown in Fig. 16. The tree contribution comes from $b \to u\bar{u}d$ transition, while there are penguin contributions with three different CKM combinations

$$\overline{A}_T = V_{ub}V_{ud}^*\, T_{u\bar{u}d}\,, \qquad \overline{A}_P = V_{tb}V_{td}^*\, P_t + V_{cb}V_{cd}^*\, P_c + V_{ub}V_{ud}^*\, P_u\,. \tag{87}$$

The convention is to rewrite the penguin contributions in terms of $V_{ub}V_{ud}^*$ and $V_{tb}V_{td}^*$ [instead of $V_{cb}V_{cd}^*$, as in Eq. (84)] using CKM unitarity as

$$\begin{aligned}\overline{A} &= V_{ub}V_{ud}^*\,(T_{u\bar{u}d} + P_u - P_c) + V_{tb}V_{td}^*\,(P_t - P_c)\\ &\equiv V_{ub}V_{ud}^*\,T + V_{tb}V_{td}^*\,P\,.\end{aligned} \tag{88}$$

where the second line defines T and P. If the penguin contribution was small, then the CP asymmetry in $B \to \pi^+\pi^-$ would measure $\mathrm{Im}\lambda_{\pi\pi}^{(\mathrm{tree})} = \sin 2\alpha$, since

$$\lambda_{\pi\pi}^{(\mathrm{tree})} = \left(\frac{V_{tb}^*V_{td}}{V_{tb}V_{td}^*}\right)\left(\frac{V_{ub}V_{ud}^*}{V_{ub}^*V_{ud}}\right) = e^{2i\alpha}\,. \tag{89}$$

The first term is the SM value of q/p in B_d mixing and the second one is $\overline{A}_T/A_T$.

The crucial new complication compared to $B \to \psi K_S$ is that the CKM elements multiplying both contributions in Eq. (88) are of order λ^3, and so $|(V_{tb}V_{td}^*)/(V_{ub}V_{ud}^*)| =$

$\mathcal{O}(1)$, whereas the analogous ratio in $B \to \psi K_S$ in Eq. (43) was $|(V_{ub}V_{us}^*)/(V_{cb}V_{cs}^*)| \simeq 1/50$. Therefore, we do not know whether amplitudes with one weak phase dominate, and our inability to model independently compute P/T results in a sizable uncertainty in the relation between $\mathrm{Im}\lambda_{\pi\pi}$ and $\sin 2\alpha$. If there are two comparable amplitudes with different weak and strong phases, then sizable CP violation in the $B \to \pi^+\pi^-$ decay is possible in addition to that in the interference between mixing and decay.

Present estimates of $|P/T|$ are around $0.2 - 0.4$. The large $B \to K\pi$ decay rate, which is probably dominated by the $b \to s$ penguin amplitudes, implies the crude estimate $|P/T| \sim \lambda \sqrt{\mathcal{B}(B \to K\pi)/\mathcal{B}(B \to \pi\pi)} \sim 0.3$, i.e., $|P/T| \not\ll 1$. The BABAR and BELLE measurements[125] do not yet show a consistent picture. BELLE measured a large value for $C_{\pi\pi}$ [see the definition in Eq. (40)], while the BABAR result is consistent with zero. If $C_{\pi\pi}$ is sizable, that implies model independently that $|P/T|$ cannot be small. However, if $C_{\pi\pi}$ is small, that may be due to a small strong phase between the P and T amplitudes and does not imply model independently that $|P/T|$ is small, nor that $S_{\pi\pi}$ is close to $\sin 2\alpha$. The central value of the BELLE measurement indicates that both the magnitude and phase of P/T has to be large, whereas the BABAR central value is consistent with a modest $|P/T|$.

There are two possible ways to deal with a non-negligible penguin contribution: (i) eliminate P (see the next section); or (ii) attempt to calculate P (see Sec. 3.4).

3.1.1 Isospin analysis

Isospin is an approximate, global $SU(2)$ symmetry of the strong interactions, violated by effects of order $(m_d - m_u)/(4\pi f_\pi) \sim 1\%$. It allows the separation of tree and penguin contributions.[126] Let's see how this works. The (u, d) quarks and the $(\bar{d}, \bar{u})$ antiquarks each form an isospin doublet, while all other (anti)quarks are singlets under $SU(2)$ isospin. Gluons couple equally to all quarks so they are also singlets. The γ and the Z are mixtures of $I = 0$ and 1, as they have unequal couplings to $u\bar{u}$ and $d\bar{d}$.

The transformation of B mesons are determined by their flavor quantum numbers, i.e., $(\overline{B}^0, B^-)$ form an $I = \frac{1}{2}$ doublet. The pions form an $I = 1$ triplet. Since the B meson and the pions are spinless particles, the pions in $B \to \pi\pi$ decay must be in a state with zero angular momentum. Because of Bose statistics, the pions have to be in an even isospin state. While $|\pi^0\pi^0\rangle$ is manifestly symmetric, when writing $|\pi^+\pi^-\rangle$ and $|\pi^0\pi^-\rangle$ what is actually meant is the symmetrized combinations $(|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle)/\sqrt{2}$ and $(|\pi^0\pi^-\rangle + |\pi^-\pi^0\rangle)/\sqrt{2}$, respectively. The isospin decompositions of $|\pi^0\pi^0\rangle$ and $|\pi^+\pi^-\rangle$ were given in Eq. (24), so we only need in addition

$$|\pi^0\pi^-\rangle = |(\pi\pi)_{I=2}\rangle\,. \tag{90}$$

The $b \to u\bar{u}d$ Hamiltonian is a mixture of $I = \frac{1}{2}$ and $\frac{3}{2}$. More precisely, it has $|I, I_z\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$ and $|\frac{3}{2}, -\frac{1}{2}\rangle$ pieces, which can only contribute to the $I = 0$ and

$I = 2$ final states, respectively. The crucial point is that the penguin operators ($O_3 - O_6$ and O_8) only contribute to the $|I, I_z\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$ part of the Hamiltonian, because the gluon is isosinglet (these operators involve a flavor sum, $\sum q'\bar{q}'$). If we can (effectively) isolate CP violation in the $I = 2$ final state then the resulting asymmetry would determine $\sin 2\alpha$. However, electroweak penguin operators [O_7 and $O_7^{\rm ew} - O_{10}^{\rm ew}$ in Eq. (86)] contribute to both $I = \frac{1}{2}$ and $\frac{3}{2}$ pieces of the Hamiltonian, and their effects cannot be separated from the tree contributions via the isospin analysis.

Besides the decomposition of the $\pi\pi$ final state in Eqs. (24) and (90), we also have to consider the combination of the $\overline{B}{}^0$ and B^- with the Hamiltonian, where another Clebsch-Gordan coefficient enters. The $I = \frac{1}{2}$ part of the Hamiltonian only contributes to $\overline{B}{}^0$ decay. However, the $I = \frac{3}{2}$ part has different matrix elements in $\overline{B}{}^0$ and B^- decay: $\langle\overline{B}{}^0|H_{I=3/2}|(\pi\pi)_{I=2}\rangle = (1/\sqrt{2})\mathcal{A}_2$, while $\langle B^-|H_{I=3/2}|(\pi\pi)_{I=2}\rangle = (\sqrt{3}/2)\mathcal{A}_2$. Thus the $A_2 \equiv \mathcal{A}_2/\sqrt{2}$ amplitude in $\overline{B}{}^0$ decay has to be multiplied by $\sqrt{3/2}$ to get the relative normalization of the B^- amplitude right. We thus obtain

$$\begin{aligned}
\overline{A}{}^{00} &\equiv A(\overline{B}{}^0 \to \pi^0\pi^0) = -\sqrt{\frac{1}{3}}\, A_0 + \sqrt{\frac{2}{3}}\, A_2\,, \\
\overline{A}{}^{+-} &\equiv A(\overline{B}{}^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}\, A_0 + \sqrt{\frac{1}{3}}\, A_2\,, \\
\overline{A}{}^{0-} &\equiv A(B^- \to \pi^0\pi^-) = \sqrt{\frac{3}{2}}\, A_2\,.
\end{aligned} \tag{91}$$

This implies the triangle relation:

$$\frac{1}{\sqrt{2}}\,\overline{A}{}^{+-} + \overline{A}{}^{00} = \overline{A}{}^{0-}\,. \tag{92}$$

Similar isospin decompositions hold for B^0 and B^+ decays, yielding another triangle relation

$$\frac{1}{\sqrt{2}}\, A^{+-} + A^{00} = A^{0+}\,. \tag{93}$$

Since only a single isospin amplitude contributes to $\overline{A}{}^{0-}$ and A^{0+}, we have $|\overline{A}{}^{0-}| = |A^{0+}|$ (however, in general, $|\overline{A}{}^{+-}| \neq |A^{+-}|$ and $|\overline{A}{}^{00}| \neq |A^{00}|$). So one can superimpose the two triangles by introducing $\widetilde{A}{}^{ij} \equiv e^{-2i\phi_{\rm T}}\overline{A}{}^{ij}$, where $\phi_T = \arg(V_{ub}V_{ud}^*)$.

Measuring the six decay rates entering Eqs. (92) and (93) allows the construction of the two triangles shown in Fig. 17. Measuring in addition the time dependent CP asymmetry in $B \to \pi^+\pi^-$ determines

$$\operatorname{Im}\lambda_{\pi^+\pi^-} = \operatorname{Im}\bigg(e^{2i\alpha}\,\frac{\widetilde{A}{}^{+-}}{A^{+-}}\bigg) = \operatorname{Im} e^{2i(\alpha+\delta)}\,. \tag{94}$$

Since δ, the strong phase difference between $\overline{A}{}^{+-}$ and A^{+-}, is known from the construction in Fig. 17, this provides a theoretically clean determination of the CKM angle

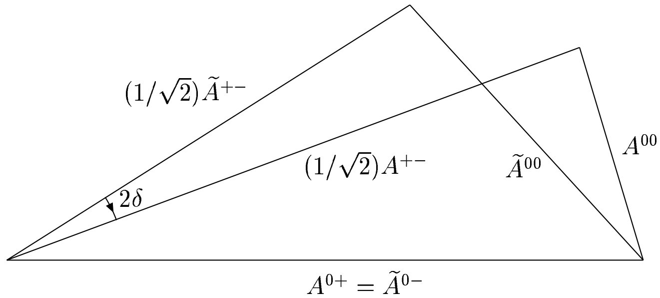


Fig. 17. The isospin triangles of Eqs. (92) and (93).

α. Probably the dominant remaining theoretical uncertainty is due to electroweak penguins mentioned above, that cannot be eliminated with the isospin analysis. This has been estimated to give a $\lesssim 5\%$ uncertainty.[4] There is also a four-fold discrete ambiguity in δ corresponding to reflections of each of the two triangles along the A^{0+} side.

A similar analysis is also possible in $B \to \rho\pi$ decays. A complication is that the final state contains non-identical particles, so it can have $I = 0$, 1, and 2 pieces. Then there are four amplitudes, and one obtains pentagon relations[127] instead of the $B \to \pi\pi$ triangle relations. It may be experimentally more feasible to do a Dalitz plot analysis that allows in principle to eliminate the hadronic uncertainties due to the QCD penguin contributions by considering only the $\pi^+\pi^-\pi^0$ final state.[128]

3.2 Some future clean measurements

We discuss below a few theoretically clean measurements that may play important roles in overconstraining the CKM picture (in addition to $B \to \phi K_S$ discussed in Sec. 1.6.3, and $B \to \pi\pi$ [$\rho\pi$] with isospin [Dalitz plot] analysis discussed above). These also indicate the complementarity between high statistics e^+e^- and hadronic B factories.

3.2.1 $B_s \to \psi\phi$ and $B_s \to \psi\eta^{(\prime)}$

Similar to $B \to \psi K_{S,L}$, the CP asymmetry in $B_s \to \psi\phi$ measures the phase difference between B_s mixing and $b \to c\bar{c}s$ decay, β_s, in a theoretically clean way. The greater than 10% CL range of $\sin 2\beta_s$ in the SM is[39] $0.026 < \sin 2\beta_s < 0.048$ (see Fig. 18).

The $\psi\phi$ final state is not a pure CP eigenstates, but it has CP self conjugate particle content and can be decomposed into CP-even and odd partial waves. An angular analysis can separate the various components, and may provide theoretically clean information on β_s. Even before this can be done, one can search for new physics, since the asymmetry measured without the angular analysis can only be smaller in magnitude than $\sin 2\beta_s$. If α^2 is the CP-even fraction of the $\psi\phi$ final state (i.e.,

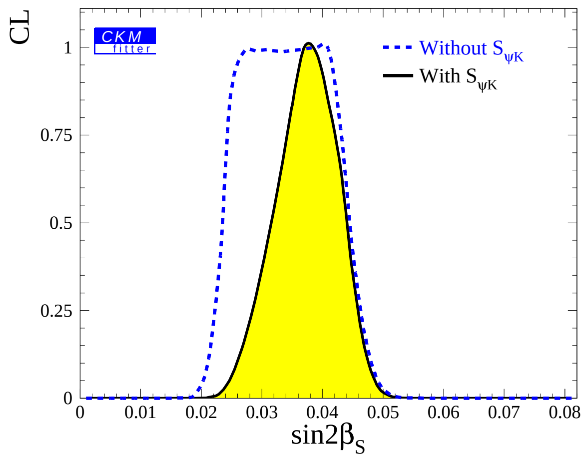


Fig. 18. Confidence levels of $\sin 2\beta_s$ in the SM with and without including the constraint from the CP asymmetry in $B \to \psi K_S$ (from Ref. [39]).

$|\psi\phi\rangle = \alpha\,|CP = +\rangle + \sqrt{1-\alpha^2}\,|CP = -\rangle$), then $S_{\psi\phi} = (2\alpha^2 - 1)\sin 2\beta_s$. Thus, the observation of a large asymmetry would be a clear signature of new physics.

The advantage of $B_s \to \psi\eta^{(\prime)}$ compared to $B_s \to \psi\phi$ is that the final states are pure CP-even. BTeV will be well-suited to measure the CP asymmetries in such modes.

3.2.2 $B_s \to D_s^{\pm} K^{\mp}$ and $B_d \to D^{(*)\pm}\pi^{\mp}$

In certain decays to final states which are not CP eigenstates, it is still possible to extract weak phases model independently from the interference between mixing and decay. This occurs if both B^0 and $\overline{B}^0$ can decay into a final state and its CP conjugate, but there is only one contribution to each decay amplitude. In such a case no assumption about hadronic physics is needed, even though $|\overline{A}_f/A_f| \neq 1$ and $|\overline{A}_{\overline{f}}/A_{\overline{f}}| \neq 1$.

An important decay of this type is $B_s \to D_s^{\pm} K^{\mp}$, which allows a model independent determination of the angle γ.[129] Both $\overline{B}{}_s^0$ and B_s^0 can decay to $D_s^+ K^-$ and $D_s^- K^+$, but there is only one amplitude in each decay corresponding to the tree level $b \to c\bar{u}s$ and $b \to u\bar{c}s$ transitions, and their CP conjugates. There are no penguin contributions to these decays. One can easily see that

$$\frac{\overline{A}_{D_s^+ K^-}}{A_{D_s^+ K^-}} = \frac{A_1}{A_2}\left(\frac{V_{cb}V_{us}^*}{V_{ub}^*V_{cs}}\right), \qquad \frac{\overline{A}_{D_s^- K^+}}{A_{D_s^- K^+}} = \frac{A_2}{A_1}\left(\frac{V_{ub}V_{cs}^*}{V_{cb}^*V_{us}}\right), \tag{95}$$

where the ratio of hadronic amplitudes, A_1/A_2, includes the strong (but not the weak) phases, and is an unknown complex number that is expected to be of order unity. It is important for the feasibility of this method that $|V_{cb}V_{us}|$ and $|V_{ub}V_{cs}|$ are both of order λ^3, and so are comparable in magnitude. Measuring the four time dependent decay rates determine $\lambda_{D_s^+ K^-}$ and $\lambda_{D_s^- K^+}$. The ratio of unknown hadronic amplitudes, A_1/A_2, drops out from their product,

$$\lambda_{D_s^+ K^-}\, \lambda_{D_s^- K^+} = \left(\frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*}\right)^2 \left(\frac{V_{cb} V_{us}^*}{V_{ub}^* V_{cs}}\right) \left(\frac{V_{ub} V_{cs}^*}{V_{cb}^* V_{us}}\right) = e^{-2i(\gamma - 2\beta_s - \beta_K)} \,. \tag{96}$$

The first factor is the Standard Model value of q/p in B_s mixing. The angles β_s and β_K defined in Eq. (18) occur in "squashed" unitarity triangles; β_s is of order λ^2 and β_K is of order λ^4. Thus we can get a theoretically clean measurement of $\gamma - 2\beta_s$.

In analogy with the above, the time dependent $B_d \to D^{(*)\pm}\pi^\mp$ rates may be used to measure $\gamma + 2\beta$, since $\lambda_{D^+\pi^-}\, \lambda_{D^-\pi^+} = \exp\,[-2i(\gamma + 2\beta)]$. In this case, however, the ratio of the two decay amplitudes is of order λ^2, and therefore the CP asymmetries are expected to be much smaller, at the percent level, making this measurement in B_d decays rather challenging.

3.2.3 $B^\pm \to (D^0, \overline{D}^0) K^\pm$ and γ

Some of the theoretically cleanest determinations of the weak phase γ rely on $B \to DK$ and related decays. The original idea of Gronau and Wyler was to measure two rates arising from $b \to c\bar{u}s$ and $b \to u\bar{c}s$ amplitudes, and a third one that involves their interference.[130] Thus one can gain sensitivity to the weak phase between the two amplitudes, which is γ in the usual phase convention. Assuming that there is no CP violation in the D sector (which is a very good approximation in the SM), and defining the CP-even and odd states as

$$|D_\pm^0\rangle = \frac{1}{\sqrt{2}} \Big(|D^0\rangle \pm |\overline{D}^0\rangle\Big), \tag{97}$$

imply the following amplitude relations,

$$\begin{aligned} \sqrt{2}\, A(B^+ \to K^+ D_+^0) &= A(B^+ \to K^+ D^0) + A(B^+ \to K^+ \overline{D}^0)\,, \\ \sqrt{2}\, A(B^- \to K^- D_+^0) &= A(B^- \to K^- D^0) + A(B^- \to K^- \overline{D}^0)\,. \end{aligned} \tag{98}$$

In the first relation, for example, $B^+ \to K^+ \overline{D}^0$ is a $b \to c$ transition, $B^+ \to K^+ D^0$ is a $b \to u$ transition, and $B^+ \to K^+ D_+^0$ receives contributions from both. Then the triangle construction in Fig. 19 determines the weak phase between the $\bar{b} \to \bar{u}$ and $b \to u$ transitions, which is 2γ (in the usual phase convention). There is again a four-fold discrete ambiguity corresponding to the reflections of the triangles. Since all the quarks which appear in $B \to DK$ decays have distinct flavors, the theoretical uncertainty arises only from higher order weak interaction effects (including, possibly, $D - \overline{D}$ mixing). There are again no penguin contributions, as in Sec. 3.2.2.

In practice there are significant problems in the application of this method. Although the amplitudes in Eq. (98) are the same order in the Wolfenstein parameter, the triangles in Fig. 19 are expected to be squashed because $|V_{ub}/V_{cb}| < \lambda$ and the

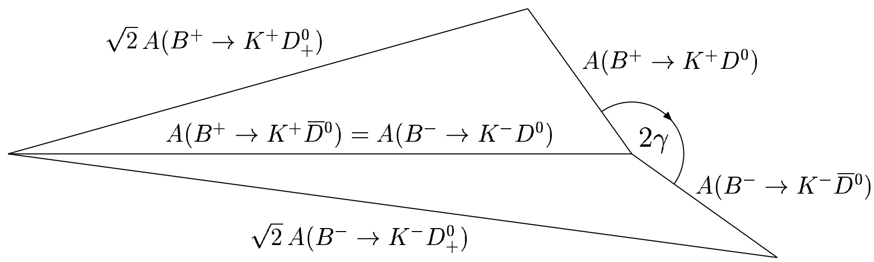


Fig. 19. Relations between $B^\pm \to DK^\pm$ amplitudes that allow determination of γ.

$B^+ \to K^+ D^0$ decay is color suppressed. The "long" sides of the triangles have been measured, including reconstruction of the D in CP eigenstates.[131] The amplitude ratio is estimated based on naive factorization as

$$\frac{|A(B^+ \to K^+ D^0)|}{|A(B^+ \to K^+ \overline{D}^0|} \sim \left|\frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*}\right| \frac{1}{N_c} \sim 0.15\,, \tag{99}$$

where $N_C = 3$ is the number of colors. As a result, the measurement of $|A(B^+ \to K^+ D^0)|$ using hadronic D decays is hampered by a significant contribution from the decay $B^+ \to K^+ \overline{D}^0$, followed by a doubly Cabibbo-suppressed decay of the $\overline{D}^0$.

This problem can be avoided by making use of large final state interactions in D decays. One can consider common final states of D^0 and $\overline{D}^0$ decay, such that[132]

$$\begin{aligned} &B^+ \to K^+ \overline{D}^0 \to K^+ f_i\,, \qquad \overline{D}^0 \to K^+ f_i \ \text{ doubly Cabibbo-suppressed}\,,\\ &B^+ \to K^+ D^0 \to K^+ f_i\,, \qquad D^0 \to K^+ f_i \ \text{ Cabibbo-allowed}\,, \end{aligned} \tag{100}$$

which reduces the difference of the magnitudes of the two interfering amplitudes. By using at least two final states (e.g., $f_1 = K^-\pi^+$ and $f_2 = K^-\rho^+$) one can determine all strong phases directly from the analysis.[132]

It may be advantageous, especially if the amplitude ratio in Eq. (99) is not smaller than its naive estimate, to consider only singly Cabibbo-suppressed D decays.[133] In this case the two final states can be $K^\pm K^{*\mp}$, corresponding to simply flipping the charge assignments, because the $D^0 \to K^+ K^{*-}$ and $D^0 \to K^- K^{*+}$ rates differ significantly. This measurement is less sensitive to $D^0 - \overline{D}^0$ mixing than considering doubly Cabibbo-suppressed D decays.[133] Moreover, all the modes that need to be measured for this method are accessible in the present data sets.

3.3 Factorization in $b \to c$ decay

Until recently little was known model independently about exclusive nonleptonic B decays. Crudely speaking, factorization is the hypothesis that, starting from the effective nonleptonic Hamiltonian, one can estimate matrix elements of four-quark operators by

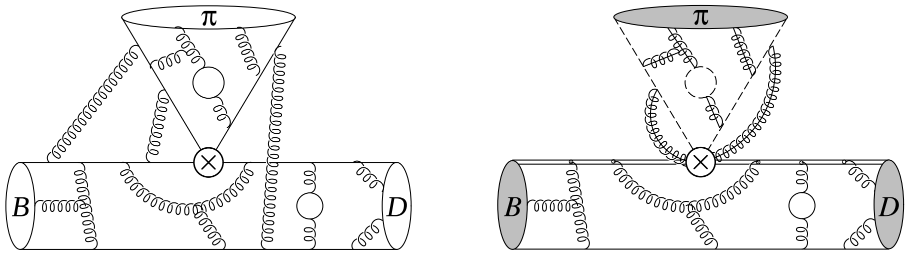


Fig. 20. Illustration of factorization in $B \to D\pi$. Left: typical diagram in full QCD. Right: typical diagram in SCET at leading order in $\Lambda_{\rm QCD}/m_Q$. The $\otimes$ denotes the weak Hamiltonian, double lines are heavy quarks, gluons with a line through them are collinear. (From Ref. [138].)

grouping the quark fields into a pair that can mediate $B \to M_1$ decay (M_1 inherits the spectator quark from the B), and another pair that can describe vacuum $\to M_2$ transition. For $M_1 = D^{(*)}$ and $M_2 = \pi$, this amounts to the assumption that the contributions of gluons between the pion and the heavy mesons are either calculable perturbatively or are suppressed by $\Lambda_{\rm QCD}/m_Q$.

It has long been known that if M_1 is heavy and M_2 is light, such as $\overline{B}^0 \to D^{(*)+}\pi^-$, then "color transparency" may justify factorization.[134–136] The physical picture is that the two quarks forming the π must emerge from the weak decay in a small (compared to $\Lambda_{\rm QCD}^{-1}$) color dipole state rapidly moving away from the D meson. At the same time the wave function of the brown muck in the heavy meson only has to change moderately, since the recoil of the D is small. While the order α_s corrections were calculated a decade ago,[136] it was only shown recently, first to 2-loops[137] and then to all orders in perturbation theory,[138] that in such decays factorization is the leading result in a systematic expansion in powers of $\alpha_s(m_Q)$ and $\Lambda_{\rm QCD}/m_Q$. The factorization formula for $\overline{B}^0 \to D^{(*)+}\pi^-$ and $B^- \to D^{(*)0}\pi^-$ decay is[137,138]

$$\langle D^{(*)}\pi|\, O_i(\mu_0)\, |B\rangle = iN_{(*)}\, F_{B\to D^{(*)}}\, f_\pi \int_0^1 {\rm d}x\, T(x,\mu_0,\mu)\, \phi_\pi(x,\mu)\,. \qquad (101)$$

where $O_{1,2}$ are the color singlet and octet operators in Eq. (83) that occur in the effective Hamiltonian in Eq. (82). Diagrams such as the one in Fig. 20 on the left give contributions suppressed by α_s or $\Lambda_{\rm QCD}/m_b$, and the leading contributions (in $\Lambda_{\rm QCD}/m_b$) come only from diagrams such as the one in Fig. 20 on the right. At leading order, soft gluons decouple from the pion, and collinear gluons with momenta scaling as $(p^-, p^\perp, p^+) \sim (m_b, \Lambda_{\rm QCD}, \Lambda_{\rm QCD}^2/m_b)$ couple only to the hard vertex [see discussion around Eqs. (58) – (59)], giving rise to the convolution integral.

In Eq. (101), $N = (m_B^2 - m_D^2)/4$ and $N_* = m_{D^*}\,(\epsilon^*\cdot p_B)/2$ are kinematic factors. There are several nonperturbative quantities, $F_{B\to D^{(*)}}$ is the $B \to D^{(*)}$ form factor at $q^2 = m_\pi^2$ measurable in semileptonic $B \to D^{(*)}\ell\bar\nu$ decay, f_π is the pion decay constant, and ϕ_π is the pion light-cone wave function that describes the probability that one of the quarks has momentum fraction x in the pion. The $T(x, \mu_0, \mu)$ is a perturbatively calculable short distance coefficient. (Strictly speaking, T depends on a third scale, μ', that cancels the μ'-dependence of the Isgur-Wise function, $\xi(w, \mu')$, which determines $F_{B\to D^{(*)}}$.) Contrary to naive factorization, which corresponds to setting $\mu_0 = m_b$ and $T = 1$, Eq. (101) provides a consistent formulation where the scale and scheme dependences cancel order by order in α_s between the Wilson coefficients $C_i(\mu_0)$ and $T(x, \mu_0, \mu)$ in the matrix elements.

The proof of factorization applies as long as the meson that inherits the brown muck from the B meson is heavy (e.g., $D^{(*)}$, D_1, etc.) and the other is light (e.g., π, ρ, etc.). The proof does not apply to decays when the spectator quark in the B ends up in the pion, such as color suppressed decays of the type $\overline{B}^0 \to D^0\pi^0$, or color allowed decays of the type $\overline{B}^0 \to D_s^-\pi^+$. Annihilation and hard spectator contributions to all decays discussed are power suppressed if one assumes that tail ends of the wave functions behave as $(\Lambda_{\rm QCD}/m_b)^a$ with $a > 0$.

While the perturbative corrections in $T(x, \mu_0, \mu)$ are calculable, little is known from first principles about the correction suppressed by powers of $\Lambda_{\rm QCD}/m_Q$. Some possibilities to learn about their size is discussed next.

3.3.1 Tests of factorization

It is important to understand quantitatively the accuracy of factorization in different processes, and the mechanism(s) responsible for factorization and its violation. Factorization also holds in the large number of colors limit ($N_c \to \infty$ with $\alpha_s N_c =$ constant) in all $B \to M_1^- M_2^+$ type decays, with corrections suppressed by $1/N_c^2$, independent of the final mesons. If factorization is mostly a consequence of perturbative QCD, then its accuracy should depend on details of the final state, since the proof outlined in the previous section relies on M_2 being fast ($m/E \ll 1$), whereas the large-N_c argument is independent of this. It would be nice to observe deviations that distinguish between these expectations, and to understand the size of power suppressed effects.

Of the nonperturbative input needed to evaluate Eq. (101), the $B \to D^{(*)}$ form factors that enter $F_{B\to D^{(*)}}$ are measured in semileptonic $B \to D^{(*)}\ell\bar\nu$ decay, and the pion decay constant f_π is also known. The pion light-cone wave function is $\phi_\pi(x) = 6x(1-x) + \ldots$, where the corrections are not too important since these decays receive small contributions from x near 0 or 1. Thus, in color allowed decays, such as $\overline{B}^0 \to D^{(*)+}\pi^-$ and $D^{(*)+}\rho^-$, factorization has been observed to work at the 10% level. These tests get really interesting just around this level, since we would like to distinguish between corrections suppressed by $\Lambda_{\rm QCD}/m_{c,b}$ and/or $1/N_c^2$.

At the level of existing data, factorization also works in $B \to D_s^{(*)} D^{(*)}$ decays, where both particles are heavy. It will be interesting to check whether there are larger corrections to factorization in $\overline{B}^0 \to D_s^{(*)-}\pi^+$ decay than in $\overline{B}^0 \to D^{(*)+}\pi^-$, since the former is expected to be suppressed in addition to $|V_{ub}/V_{cb}|^2$ by $\Lambda_{\rm QCD}/m_{c,b}$ as well.[139,140] For this test, measurement of the $B \to \pi\ell\bar\nu$ form factor is necessary. Another test involves decays to "designer mesons", such as $\overline{B}^0 \to D^{(*)+}d^-$ (where $d = a_0, b_1, \pi_2$, etc.), which vanish in naive factorization, so the order α_s and $\Lambda_{\rm QCD}/m_{c,b}$ terms are expected to be the leading contributions.[141]

One of the simplest detailed tests of factorization is the comparison of the $\overline{B}^0 \to D^{(*)+}\pi^-$ and $B^- \to D^{(*)0}\pi^-$ rates and isospin amplitudes. These rates are predicted to be equal in the $m_{c,b} \gg \Lambda_{\rm QCD}$ limit, since they only differ by a power suppressed contribution to $B^- \to D^{(*)0}\pi^-$ when the spectator in the B ends up in the π. Let's work this out for fun in detail.

***$B \to D\pi$* isospin analysis** The initial $(\overline{B}^0, B^-)$ and final (D^+, D^0) are $I = \frac12$ doublets, the pions are in an $I = 1$ triplet. So $D\pi$ can be in $I = \frac12$ or $\frac32$ state, and the decomposition is

$$\begin{aligned}
|D^0\pi^0\rangle &= -\sqrt{\frac13}\,|(D\pi)_{I=1/2}\rangle + \sqrt{\frac23}\,|(D\pi)_{I=3/2}\rangle\,, \\
|D^+\pi^-\rangle &= \sqrt{\frac23}\,|(D\pi)_{I=1/2}\rangle + \sqrt{\frac13}\,|(D\pi)_{I=3/2}\rangle\,, \\
|D^0\pi^-\rangle &= |(D\pi)_{I=3/2}\rangle\,.
\end{aligned} \tag{102}$$

The $b \to c\bar u d$ Hamiltonian is $|I, I_z\rangle = |1, -1\rangle$. Similar to Sec. 3.1.1, we need to be careful with the relative normalization of the $\overline{B}^0$ and B^- decay matrix elements. The $I = \frac12$ amplitude only occurs in $\overline{B}^0$ decay, and there is no subtlety. The $I = \frac32$ amplitude occurs with different normalization in neutral and charged B decay: $\langle \overline{B}^0|H|(D\pi)_{I=3/2}\rangle = (1/\sqrt3)\mathcal{A}_{3/2}$, while $\langle B^-|H|(D\pi)_{I=3/2}\rangle = \mathcal{A}_{3/2}$. Thus the $A_{3/2} \equiv \mathcal{A}_{3/2}/\sqrt3$ amplitude in $\overline{B}^0$ decay needs to be multiplied by $\sqrt3$ to get the normalization of the B^- decay amplitude right. We get

$$\begin{aligned}
A^{00} \equiv A(\overline{B}^0 \to D^0\pi^0) &= -\sqrt{\frac13}\,A_{1/2} + \sqrt{\frac23}\,A_{3/2}\,, \\
A^{+-} \equiv A(\overline{B}^0 \to D^+\pi^-) &= \sqrt{\frac23}\,A_{1/2} + \sqrt{\frac13}\,A_{3/2}\,, \\
A^{0-} \equiv A(B^- \to D^0\pi^-) &= \sqrt3\,A_{3/2}\,.
\end{aligned} \tag{103}$$

This implies the triangle relation:

$$A^{+-} + \sqrt2\,A^{00} = A^{0-}\,. \tag{104}$$

A prediction of QCD factorization in $B \to D\pi$ decay is that amplitudes involving the spectator quark in the B going into the π should be power suppressed,[137,138] and therefore,

$$\frac{|A^{0-}|}{|A^{+-}|} = 1 + \mathcal{O}\Big(\frac{\Lambda_{\rm QCD}}{m_{c,b}}\Big)\,, \tag{105}$$

or in terms of isospin amplitudes, $A_{1/2} = \sqrt{2}\,A_{3/2}\,[1 + \mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})]$. In this case the triangle in Eq. (104) becomes squashed, and the strong phase difference between the $A_{1/2}$ and $A_{3/2}$ amplitudes is suppressed, $\delta_{1/2} - \delta_{3/2} = \mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$. The experimental data are[142,143]

$$\frac{\mathcal{B}(B^- \to D^0\pi^-)}{\mathcal{B}(\overline{B}{}^0 \to D^+\pi^-)} = 1.85 \pm 0.25\,,$$
$$16.5^\circ < \delta_{1/2} - \delta_{3/2} < 38.1^\circ \quad (90\%\ {\rm CL})\,. \tag{106}$$

The ratio of branching ratios is measured to be in the ballpark of 1.8 also for D replaced by D^* and π replaced by ρ. These deviations from factorization are usually attributed to $\mathcal{O}(\Lambda_{\rm QCD}/m_c)$ corrections,[137] which could be of order 30% in the amplitudes and twice that in the rates. One could claim that the strong phase in Eq. (106) should be viewed as small, since $1 - \cos 26^\circ \simeq 0.1 \ll 1$. This is open to interpretation, as the answer depends sensitively on the measure used (and, for example, we think of the CKM angle $\beta \approx 23.5^\circ$ as order unity).

Studying such two-body channels it is hard to unambiguously identify the source of the corrections to factorization. The problem is that the color suppressed contribution to the $B^- \to D^0\pi^-$ is formally order $1/N_c$ in the large N_c limit, and order $\Lambda_{\rm QCD}/m_{c,b}$ in the heavy mass limit, which may be comparable. Factorization fails even worse in $D \to K\pi$ decays, however this does not show model independently that the corrections seen in $B \to D\pi$ are due to $\Lambda_{\rm QCD}/m_Q$ effects, since the proof of factorization based on the heavy quark limit does not apply for $D \to K\pi$ to start with. It does indicate, however, that the large N_c limit cannot be the full story.

Factorization in $B \to D^{(*)}X$ Another possibility to study corrections to factorization is to consider $B \to D^{(*)}X$ decay where X contains two or more hadrons. The advantage compared to two-body channels is that the accuracy of factorization can be studied as a function of kinematics for final states with fixed particle content, by examining the differential decay rate as a function of the invariant mass of the light hadronic state X.[144,145,135] If factorization works primarily due to the large N_c limit, then its accuracy is not expected to decrease as the invariant mass of X, m_X, increases. However, if factorization is mostly a consequence of perturbative QCD, then the corrections should grow with m_X. Factorization has also been studied in inclusive $B \to D^{(*)}X$ decay, and it was suggested that the small velocity limit ($m_b,\ m_c \gg m_b - m_c \gg \Lambda_{\rm QCD}$) may also play an important role in factorization.[146]

Combining data for hadronic τ decay (which effectively measures the hadronization of a virtual W to X) and semileptonic B decay allows such tests to be made for a variety

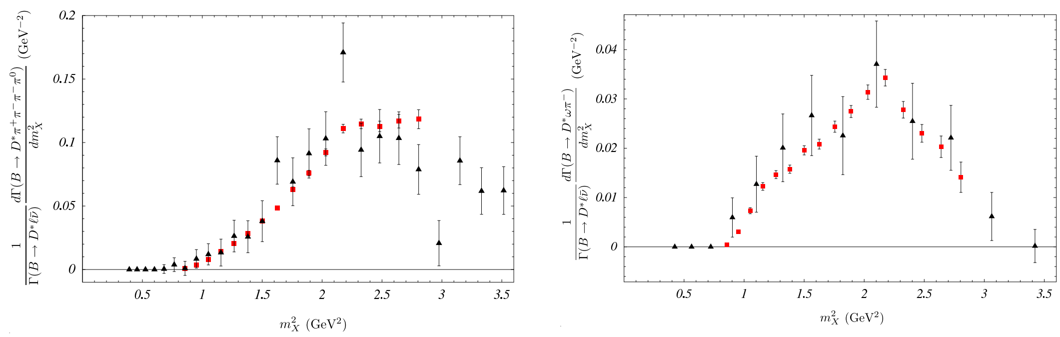


Fig. 21. $\mathrm{d}\Gamma(B \to D^*X)/\mathrm{d}m_X^2$, where $X = \pi^+\pi^-\pi^-\pi^0$ (left) and $X = \omega\pi$ (right), normalized to the semileptonic width $\Gamma(B \to D^*\ell\bar{\nu})$. The triangles are B decay data§ and the squares are the predictions using τ data. (From Ref. [144].)

of final states. Figure 21 shows the comparison of the $B \to D^*\pi^+\pi^-\pi^-\pi^0$ and $D^*\omega\pi^-$ data[147] with the τ decay[148] data. The reason to consider the 4π final state is because the 2π and 3π channels are dominated by resonances. The kinematic range accessible in $\tau \to 4\pi$ corresponds to $0.4 \lesssim m_{4\pi}/E_{4\pi} \lesssim 0.7$ in $B \to 4\pi$ decay. A background to these comparisons is that one or more of the pions may arise from the $\bar{c}_L\gamma^\mu b_L$ current instead of the $\bar{d}_L\gamma^\mu u_L$ current. In the $\omega\pi^-$ mode this is very unlikely to be significant.[144] In the $\pi^+\pi^-\pi^-\pi^0$ mode such backgrounds can be constrained by measuring the $B \to D^*\pi^+\pi^+\pi^-\pi^-$ rate, since $\pi^+\pi^+\pi^-\pi^-$ cannot come from the $\bar{d}_L\gamma^\mu u_L$ current. CLEO found $\mathcal{B}(B \to D^*\pi^+\pi^+\pi^-\pi^-)/\mathcal{B}(B \to D^*\pi^+\pi^-\pi^-\pi^0) < 0.13$ at 90% CL in the $m_X^2 < 2.9\,\mathrm{GeV}^2$ region,[149] consistent with zero. When more precise data are available, observing deviations that grow with m_X would be evidence that perturbative QCD is an important part of the success of factorization in $B \to D^*X$.

3.4 Factorization in charmless B decays

Calculating B decay amplitudes to charmless two-body final states is especially important for the study of CP violation. There are two contributions to these decays shown schematically in Fig. 22. The first term is analogous to the leading term in $\overline{B}^0 \to D^+\pi^-$, while the second one involves hard spectator interaction. There are two approaches to factorization in these decays, which differ even on the question of which of the two contributions is the leading one in the heavy quark limit.

§In this case the charged and neutral B decay rates do not differ significantly, $\mathcal{B}(B^- \to D^{*0}\pi^+\pi^-\pi^-\pi^0) = (1.80 \pm 0.36)\%$ and $\mathcal{B}(\overline{B}^0 \to D^{*+}\pi^+\pi^-\pi^-\pi^0) = (1.72 \pm 0.28)\%$.[147] Their ratio is certainly smaller then the similar ratio in Eq. (106), typical for $B \to D^{(*)}\pi$ and $D^{(*)}\rho$ decays. In addition, $\mathcal{B}(\overline{B}^0 \to D^{*0}\pi^+\pi^+\pi^-\pi^-) = (0.30 \pm 0.09)\%$ is small[149] and sensitive to contributions when the spectator in the B does not end up in the D^*.

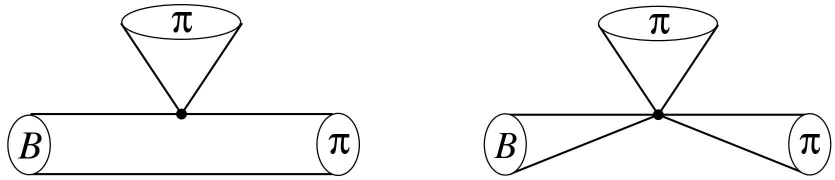


Fig. 22. Two contributions to $B \to \pi\pi$ amplitudes (from Ref. [64]).

Beneke *et al.* (BBNS)[150] proposed a factorization formula

$$\langle \pi\pi | O_i | B \rangle = F_{B\to\pi} \int \mathrm{d}x \, T^I(x) \, \phi_\pi(x) + \int \mathrm{d}\xi \, \mathrm{d}x \, \mathrm{d}x' \, T^{II}(\xi, x, x') \, \phi_B(\xi) \, \phi_\pi(x) \, \phi_\pi(x') \,, \tag{107}$$

and showed that it is consistent to first order in α_s. The T's are calculable short distance coefficient functions, whereas the ϕ's are nonperturbative light-cone distribution functions. Each of these terms have additional scale dependences not shown above, similar to those in Eqs. (60) and (101), which are supposed to cancel order-by-order in physical results. A major complication of charmless decays compared $B \to D\pi$ is that understanding the role of endpoint regions of the light-cone distribution functions is much more involved. BBNS assume that Sudakov suppression is not effective at the B mass scale in the endpoint regions of these distribution functions. Then the two terms are of the same order in $\Lambda_{\rm QCD}/m_b$, but the second term is suppressed by α_s.

Keum *et al.* (KLS)[151] assume that Sudakov suppression is effective in suppressing contributions from the tails of the wave functions, $x \sim \Lambda_{\rm QCD}/m_b$. Then the first term [in Eq. (107) and in Fig. 22] is subleading and the second one gives the dominant contribution. This issue is related to Sec. 2.2.2, where an open question was the relative size of the two contributions to the $B \to \pi\ell\bar\nu$ form factors in Eq. (60). These form factors are calculable according to KLS (in terms of the poorly known B and π light-cone wave functions), whereas they are nonperturbative inputs that can only be determined from data according to BBNS.

The outstanding open theoretical questions are to prove the factorization formula to all orders in α_s (this was claimed very recently[152]), to understand the role of Sudakov effects, and to find out which contribution (if either) is dominant in the heavy mass limit. Before these questions are answered, it is not clear that either approach is right. A complete formulation of power suppressed corrections is also lacking so far.

Some terms that are formally order $\Lambda_{\rm QCD}/m_b$ in the BBNS approach are known to be large numerically and must be included to be able to describe the data. These are the so-called "chirally enhanced" terms proportional to $m_K^2/(m_s m_b)$, which are actually not enhanced by any parameter of QCD in the chiral limit, they are just $\mathcal{O}(\Lambda_{\rm QCD})$, but happen to be large. The uncertainty related to weak annihilation contributions also needs to be better understood. Note that diagrams usually called annihilation cannot be

Experimental Observable	Theoretical BBNS	Predictions KLS	World Average
$\dfrac{\mathcal{B}(\pi^+\pi^-)}{\mathcal{B}(\pi^\mp K^\pm)}$	$0.3-1.6$	$0.3-0.7$	0.28 ± 0.04
$\dfrac{\mathcal{B}(\pi^\mp K^\pm)}{2\,\mathcal{B}(\pi^0 K^0)}$	$0.9-1.4$	$0.8-1.05$	1.0 ± 0.3
$\dfrac{2\,\mathcal{B}(\pi^0 K^\pm)}{\mathcal{B}(\pi^\pm K^0)}$	$0.9-1.3$	$0.8-1.6$	1.3 ± 0.2
$\dfrac{\tau_{B^\pm}}{\tau_{B^0}}\,\dfrac{\mathcal{B}(\pi^\mp K^\pm)}{\mathcal{B}(\pi^\pm K^0)}$	$0.6-1.0$	$0.7-1.45$	1.1 ± 0.1
$\dfrac{\tau_{B^\pm}}{\tau_{B^0}}\,\dfrac{\mathcal{B}(\pi^+\pi^-)}{2\,\mathcal{B}(\pi^\pm\pi^0)}$	$0.6-1.1$		0.56 ± 0.14

Table 5. Experimental data and theoretical predictions/postdictions for ratios of $B\to\pi\pi, K\pi$ branching ratios (from Ref. [42]).

distinguished from rescattering. The $B^0\to D_sK$ data[153] seems to indicate that these are not very strongly suppressed.

3.4.1 Phenomenology of $B\to\pi\pi, K\pi$

While the two approaches discussed above yield different power counting and sometimes different phenomenological predictions, so far the results from both groups fit (or could be adjusted to fit) the data on charmless two-body B decays. It has also been claimed that the effects of charm loops are larger than predicted by either approach.[154] Table 5 compares theory and data for ratios of certain charmless B decay rates. Conclusive tests do not seem easy, and it will take a lot of data to learn about the accuracy of these predictions. Predictions for strong phases and therefore for direct CP violation are typically smaller in BBNS than in the KLS approach. More precise experimental data will be crucial.

A CKM fit assuming BBNS and using the $B\to\pi\pi, K\pi$ rates and direct CP asymmetries is shown in Fig. 23. It yields a $\rho-\eta$ region consistent with the "standard" CKM fits, although preferring slightly larger values of γ. Similar results might be obtained using the KLS predictions as inputs. A recent analysis including pseudoscalar–vector modes as well finds an unsatisfactory fit to the data.[156] Note that if the lattice results for $\xi^2\equiv(f_{B_s}^2B_{B_s})/(f_{B_d}^2B_{B_d})$ increase when light quark effects are fully understood, the possibility of which was mentioned in Sec. (1.5), and if the B_s mass difference is near the present limits, that would shift the "standard" fit to somewhat larger values of γ.

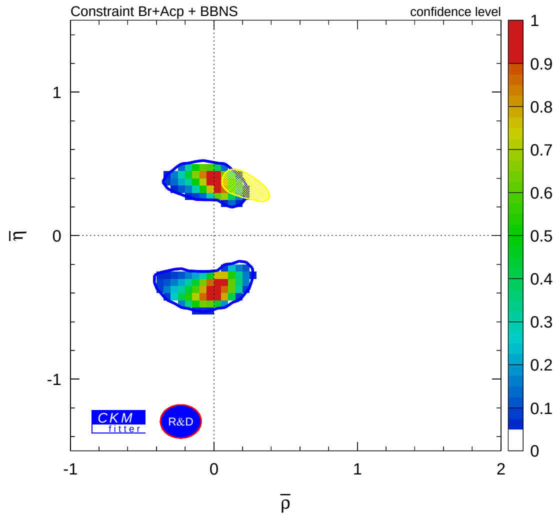


Fig. 23. Fit to charmless two-body B decays assuming BBNS (from Ref. [155]).

Many strategies have been proposed to use $SU(3)$ flavor symmetry to constrain CKM angles by combining data from several decay modes. For example, one might use a combination of B and B_s decays to $\pi\pi, K\pi, KK$ final states to gain sensitivity to γ without relying on a complete calculation of the hadronic matrix elements.[157] The basic idea is that $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related by U-spin, that exchanges $d \leftrightarrow s$. In such analyses one typically still needs some control over hadronic uncertainties that enter related to, for example, first order $SU(3)$ breaking effects (U-spin breaking is controlled by the same parameter, $m_s/\Lambda_{\chi SB}$), rescattering effects, etc. The crucial question is how experimental data can be used to set bounds on the size of these uncertainties. Such analyses will be important and are discussed in more detail in Frank Würthwein's lectures.[16]

Summary of factorization

- In nonleptonic $B \to D^{(*)}X$ decay, where X is a low mass hadronic state, factorization is established in the heavy quark limit, at leading order in $\Lambda_{\rm QCD}/m_Q$.
- Some of the order $\Lambda_{\rm QCD}/m_c$ corrections are sizable, and there is no evidence yet of factorization becoming a worse approximation in $B \to D^{(*)}X$ as m_X increases.
- In charmless nonleptonic decays there are two approaches: BBNS and KLS. Different assumptions and power counting, and sometimes different predictions.
- Progress in understanding charmless semileptonic and rare decay form factors in small q^2 region will help resolve power counting in charmless nonleptonic decay.
- New and more precise data will be crucial to test factorization and tell us about significance of power suppressed contributions in various processes.

3.5 Final remarks

The recent precise determination of $\sin 2\beta$ and other measurements make it very likely that the CKM contributions to flavor physics and CP violation are the dominant ones. The next goal is not simply to measure ρ and η, or α and γ, but to probe the flavor sector of the SM to high precision by many overconstraining measurements. Measurements which are redundant in the SM but sensitive to different short distance physics are very important, since correlations may give additional information on the possible new physics encountered (e.g., comparing $\Delta m_s/\Delta m_d$ with $\mathcal{B}(B \to X_s\ell^+\ell^-)/\mathcal{B}(B \to X_d\ell^+\ell^-)$ is not "just another way" to measure $|V_{ts}/V_{td}|$).

Hadronic uncertainties are often significant and hard to quantify. The sensitivity to new physics and the accuracy with which the SM can be tested will depend on our ability to disentangle the short distance physics from nonperturbative effects of hadronization. While we all want small errors, the history of ϵ'_K reminds us to be conservative with theoretical uncertainties. One theoretically clean measurement is worth ten dirty ones. But what is considered theoretically clean changes with time, and there is significant progress toward understanding the hadronic physics crucial both for standard model measurements and for searches for new physics. For example, for (i) the determination of $|V_{ub}|$ from inclusive B decay; (ii) understanding exclusive rare decay form factors at small q^2; and (iii) establishing factorization in certain nonleptonic decays.

In testing the SM and searching for new physics, our understanding of CKM parameters and hadronic physics will have to improve in parallel. Except for a few clean cases (like $\sin 2\beta$) the theoretical uncertainties can be reduced by doing several measurements, or by gaining confidence about the accuracy of theoretical assumptions. Sometimes data may help to constrain or get rid of nasty things hard to know model independently (e.g., excited state contributions to certain processes).

With the recent spectacular start of the B factories an exciting era in flavor physics has begun. The precise measurements of $\sin 2\beta$ together with the sides of the unitarity triangle, $|V_{ub}/V_{cb}|$ at the e^+e^- B factories and $|V_{td}/V_{ts}|$ at the Tevatron, will allow us to observe small deviations from the Standard Model. The large statistics will allow the study of rare decays and to improve sensitivity to observables which vanish in the SM; these measurements have individually the potential to discover physics beyond the SM. If new physics is seen, then a broad set of measurements at both e^+e^- and hadronic B factories and $K \to \pi\nu\bar{\nu}$ may allow to discriminate between classes of models. It is a vibrant theoretical and experimental program, the breadth of which is well illustrated by the long list of important measurements where significant progress is expected in the next couple of years:

- $|V_{td}/V_{ts}|$: the Tevatron should nail it, hopefully soon — will all the lattice subtleties be reliably understood by then?

- β: reduce error in ϕK_S, $\eta' K_S$, and KKK modes — will the difference from $S_{\psi K}$ become more significant?
- β_s: is CP violation in $B_s \to \psi\phi$ indeed small, as predicted by the SM?
- Rare decays: $B \to X_s\gamma$ near theory limited; more precise data on q^2 distribution in $B \to X_s\ell^+\ell^-$ will be interesting.
- $|V_{ub}|$: reaching $< 10\%$ would be important. Need to better understand $|V_{cb}|$ as well; could be a BABAR/BELLE measurement unmatched by LHCB/BTeV.
- α: Is the $\pi^+\pi^-\pi^0$ Dalitz plot analysis feasible — are there significant resonances in addition to $\rho\pi$? How small are $\mathcal{B}(B \to \pi^0\pi^0)$ and $\mathcal{B}(B \to \rho^0\pi^0)$?
- γ: the clean modes are hard — need to try all. Start to understand using data the accuracy of $SU(3)$ relations, factorization, and related approaches.
- Search for "null observables", such as $a_{CP}(b \to s\gamma)$, etc., enhancement of $B_{d,s} \to \ell^+\ell^-$, $B \to \ell\nu$, etc.

This is surely an incomplete list, and I apologize for all omissions. Any of these measurements could have a surprising result that changes the future of the field. And it is only after these that LHCB/BTeV and possibly a super-B-factory enter the stage.

3.5.1 Summary

- The CKM picture is predictive and testable — it passed its first real test and is probably the dominant source of CP violation in flavor changing processes.
- The point is not only to measure the sides and angles of the unitarity triangle, (ρ, η) and (α, β, γ), but to probe CKM by overconstraining it in as many ways as possible (large variety of rare decays, importance of correlations).
- The program as a whole is a lot more interesting than any single measurement; all possible clean measurements, both CP violating and conserving, are important.
- Many processes can give clean information on short distance physics, and there is progress toward being able to model independently interpret new observables.

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